Mandag 7. september

Rotasjon om (1,1)
med vinkel π

1 Flytt $\begin{pmatrix}1\\1\end{pmatrix}$ til origo
2 Roter med π
3 Flytt tilbake

$$\begin{pmatrix}-1&0\\0&-1\end{pmatrix}\left(\begin{pmatrix}x\\y\end{pmatrix}-\begin{pmatrix}1\\1\end{pmatrix}\right)+\begin{pmatrix}1\\1\end{pmatrix}$$

$$=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+\begin{pmatrix}1\\1\end{pmatrix}+\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+\begin{pmatrix}2\\2\end{pmatrix}$$

dvs $\underline{\underline{t_{\binom{2}{2}}S_\pi}}$

Utledning av trigonometriske likheter

$$S_\varphi S_\psi = S_{\varphi+\psi} \qquad \begin{pmatrix}\cos\varphi & -\sin\varphi\\ \sin\varphi & \cos\varphi\end{pmatrix}\begin{pmatrix}\cos\psi & -\sin\psi\\ \sin\psi & \cos\psi\end{pmatrix}=\begin{pmatrix}\cos(\varphi+\psi) & -\sin(\varphi+\psi)\\ \sin(\varphi+\psi) & \cos(\varphi+\psi)\end{pmatrix}$$

$$\cos\varphi\cos\psi - \sin\varphi\sin\psi = \cos(\varphi+\psi)$$
$$\sin\varphi\cos\psi + \cos\varphi\sin\psi = \sin(\varphi+\psi)$$

Refleksjon over linja $x-2y=2$

Normalvektor $\begin{pmatrix}1\\-2\end{pmatrix}=\bar{v}$

Flytter linja til gjennom origo $\begin{pmatrix}x\\y\end{pmatrix}-\begin{pmatrix}0\\-1\end{pmatrix}=\begin{pmatrix}x'\\y'\end{pmatrix}$

$$x'-2y'=x-2(y+1)=x-2y-2=2-2=0$$

$$Q_{\bar{v}}=\begin{pmatrix}1&0\\0&1\end{pmatrix}-2\,\frac{1}{5}\begin{pmatrix}1&-2\\-2&4\end{pmatrix}=\frac{1}{5}\begin{pmatrix}3&4\\4&-3\end{pmatrix}$$

$$\frac{1}{5}\begin{pmatrix}3&4\\4&-3\end{pmatrix}\left(\begin{pmatrix}x\\y\end{pmatrix}-\begin{pmatrix}0\\-1\end{pmatrix}\right)+\begin{pmatrix}0\\-1\end{pmatrix}=\frac{1}{5}\begin{pmatrix}3&4\\4&-3\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+\begin{pmatrix}\frac{4}{5}\\-\frac{3}{5}\end{pmatrix}+\begin{pmatrix}0\\-1\end{pmatrix}$$

$$=\frac{1}{5}\begin{pmatrix}3&4\\4&-3\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+\frac{1}{5}\begin{pmatrix}4\\-8\end{pmatrix}$$

Feks. $x=4,\ y=1$:

$$\frac{1}{5}\begin{pmatrix}3&4\\4&-3\end{pmatrix}\begin{pmatrix}4\\1\end{pmatrix}+\frac{1}{5}\begin{pmatrix}4\\-8\end{pmatrix}=\frac{1}{5}\left(\begin{pmatrix}16\\13\end{pmatrix}+\begin{pmatrix}4\\-8\end{pmatrix}\right)=\begin{pmatrix}4\\1\end{pmatrix}$$

Diskret undergruppe av $\text{Isom}_2 \supseteq G$

$L_G = \{ t_{\bar{a}} \in G \}$

$\forall\, t_{\bar{a}} \in G,\ \bar{a} \neq 0,\ \|\bar{a}\| \geq \varepsilon$

$= \{ t_{m\bar{a}+n\bar{b}} \mid m,n \in \mathbb{Z} \}$

$\forall\, \rho_{\theta,p} \in G\ \ \theta \neq 0,\ |\theta| \geq \varepsilon$

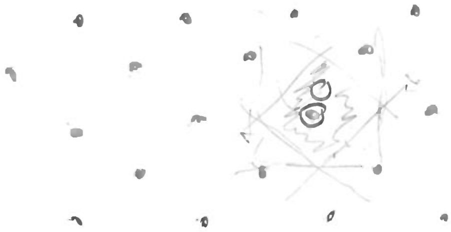
Gitter av rang 2

Mengden av gitterpunkt $M_G = \{ m\bar{a} + n\bar{b} \mid m,n \in \mathbb{Z} \}$

Vi kan anta $\bar{a} \cdot \bar{b} \geq 0$ - (spiss

Dirichletområde:

$$D_G = \{ p \in \mathbb{R}^2 \mid d(p,0) \leq d(p,q)\ \forall q \in M_G,\ q \neq 0 \}$$



Punktgruppe av $t_{\bar{a}} T$: T

Setning: Punktgruppen tar D_G på seg selv.

$g \in t_{\bar{b}} T \qquad g t_a g^{-1} = t_{\bar{b}} T t_a T^{-1} t_{-b}$

$= t_{\bar{b}}\, t_{T(\bar{a})} T T^{-1} t_{-\bar{b}}$

$T t_{\bar{a}} = t_{T(\bar{a})} T$

$= t_{T(\bar{a})}$

fordi

$T(\bar{x}+\bar{a}) = T(\bar{x}) + T(\bar{a})$

$d(T(\bar{x}), \bar{0}) = d(T(\bar{x}), T\bar{0}) = d(\bar{x}, \bar{0})$

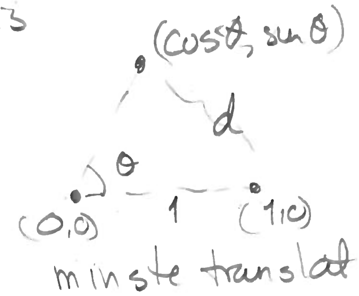
$T(M_G) = M_G$

$d(T(\bar{x}), \bar{y}) = d(T(\bar{x}), T(\bar{z})) = d(x, \bar{z}) \quad \forall\, \bar{y}, \bar{z} \in M_G$
$(\neq \bar{0})$

$\Rightarrow \{ p \in \mathbb{R}^2 \mid d(T(p), 0) \leq d(p,q) \} = \{ p \in \mathbb{R}^2 \mid d(p,0) \leq d(p,q) \}$

Setning Punktgruppen til en diskret undergruppe av Isom_2 med rg $L_G = 2$ er en av C_n, D_n men $n = 1, 2, 3, 4, 6$

Skisse av bevis



Rotasjon med vinkel $\theta = \frac{2\pi}{n}$

$$d = \sqrt{(\cos\theta - 1)^2 + (\sin\theta)^2}$$

$$= \sqrt{\cos^2\theta - 2\cos\theta + 1 + \sin^2\theta}$$

$$= \sqrt{2 - 2\cos\theta} \geq 1 \qquad \theta \neq 0$$

dvs $2 - 2\cos\theta \geq 1$

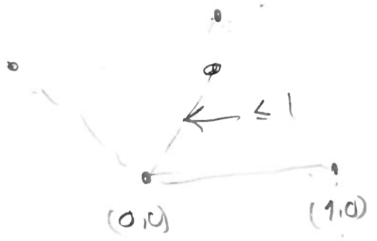
$$\Rightarrow \cos\theta \leq \frac{1}{2}$$

$$\theta = \frac{2\pi}{n} \Rightarrow \frac{2\pi}{n} \geq \frac{2\pi}{6}$$

$$\Rightarrow n \leq 6.$$

dvs. $n = 1, 2, 3, 4, 5, 6$

Men $n = 5$ gir



$$\|(\cos\tfrac{4\pi}{5}, \sin\tfrac{4\pi}{5}) + (1,0)\|^2 = \left(\cos^2\tfrac{4\pi}{5} + 2\cos\tfrac{4\pi}{5} + 1 + \sin^2\tfrac{4\pi}{5}\right)$$

$$= 2 + 2\cos\tfrac{4\pi}{5}$$

$$= 2 + 2\left(2\cos^2\tfrac{2\pi}{5} - 1\right) = 4 \cdot \cos^2\tfrac{2\pi}{5}$$

$$= 4 \cdot \left(\frac{\sqrt{5}-1}{4}\right)^2 = \frac{1}{4}(5 - 2\sqrt{5} + 1) = \frac{3-\sqrt{5}}{2} < 1.$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= \cos^2\theta - 1 + \cos^2\theta$$
$$= 2\cos^2\theta - 1$$