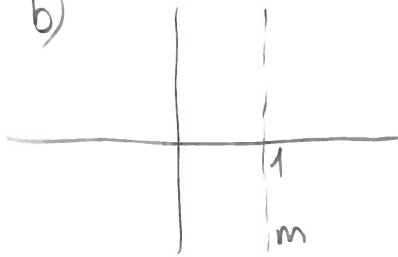


Torsdag 10. september

1.

b)

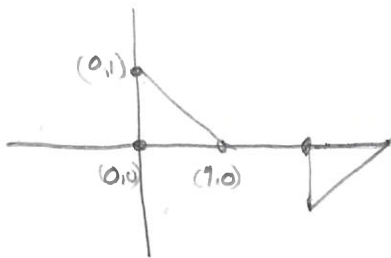


$$S_m \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

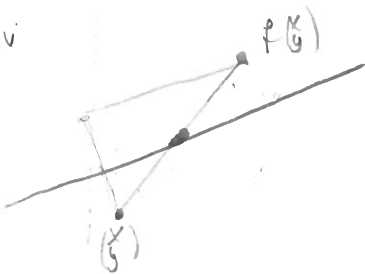
$$g = S_m \circ S_{\pi} : g \begin{pmatrix} x \\ y \end{pmatrix} = S_m \begin{pmatrix} -x \\ -y \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -x \\ -y \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} x+2 \\ -y \end{pmatrix}$$

$$g \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad g \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad g \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$



$$\Rightarrow g \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

Alternativ



$\frac{1}{2} (p(x, y) + f(x, y))$ ligger på spekuleringsslinja

$$\frac{1}{2} \left(\begin{pmatrix} 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\frac{1}{2} \left(\begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right) = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$\frac{1}{2} \left(\begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

spekuleringsslinjer: $y=0$

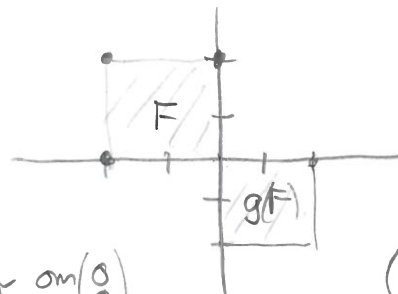
$$\vec{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \vec{v}^\perp = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$p \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} = 2 \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 0 \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \text{Gliding} \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

c)

$$F: (-2, 2) \quad (-2, 0) \quad (0, 2) \quad (0, 0)$$

$$g(F): (0, -2) \quad (0, 0) \quad (2, -2) \quad (2, 0)$$



Symmetrier: Rotasjon S_{π} om $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

Refleksjon μ om $y=x$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\text{Refleksjon } \mu \circ S_{\pi} \text{ om } y=-x \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

Begrepet geometrisk sted

En punktmengde definert av en bestemt geometrisk betingelse

- Eksempler:
- Alle punkter med en fast avstand til gitt punkt
 - Alle punkter med samme avstand til to gitte punkter
 - Toppunktet til en bestemt vinkel hvor vinkelbeina går gjennom to faste punkter
 - Alle punkter P slik at $\frac{PA}{PB} = k (> 0)$
Apollonius sirkel

$$\begin{aligned} A &= (0,0) \\ B &= (1,0) \\ P &= (x,y) \end{aligned} \quad \frac{\sqrt{x^2+y^2}}{\sqrt{(x-1)^2+y^2}} = k \Leftrightarrow x^2+y^2 = k^2(x^2-2x+1+y^2)$$

$$\Leftrightarrow (1-k^2)x^2 + 2k^2x + k^2 + (1-k^2)y^2 = 0$$

Hvis $k=1$: Multinomial

$$\text{Hvis } k \neq 1: x^2 + \frac{2k^2}{1-k^2}x + \frac{k^2}{1-k^2} + y^2 = 0$$

$$\left(x + \frac{k^2}{1-k^2}\right)^2 - \frac{k^4}{(1-k^2)^2} + \frac{k^2}{1-k^2} + y^2 = 0$$

$$\left(x + \frac{k^2}{1-k^2}\right)^2 + y^2 = \frac{2k^4 - k^2}{(1-k^2)^2}$$

sirkel

sentrum: $\left(-\frac{k^2}{1-k^2}, 0\right)$

radius: $\frac{\sqrt{2k^4 - k^2}}{(1-k^2)} = \frac{k}{1-k^2} \sqrt{2k^2 - 1}$

- Alle punkter hvor summen av avstandene til to faste punkter er konstant

$$\sqrt{x^2+y^2} + \sqrt{(x-1)^2+y^2} = k$$

$$x^2+y^2 + 2\sqrt{x^2+y^2}\sqrt{(x-1)^2+y^2} + (x-1)^2+y^2 = k^2$$

$$4(x^2+y^2)((x-1)^2+y^2) = (k^2 - x^2 - y^2 - (x-1)^2 - y^2)^2 = (k^2 - x^2 - (x-1)^2 - 2y^2)^2$$

$$\left(x - \frac{1}{2}\right)^2 + \frac{k^2}{k^2-1} y^2 = k^2-1 \quad \underline{\text{ellipse}}$$

standard form for ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

— " — hyperbel

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \pm 1$$

— " — parabel

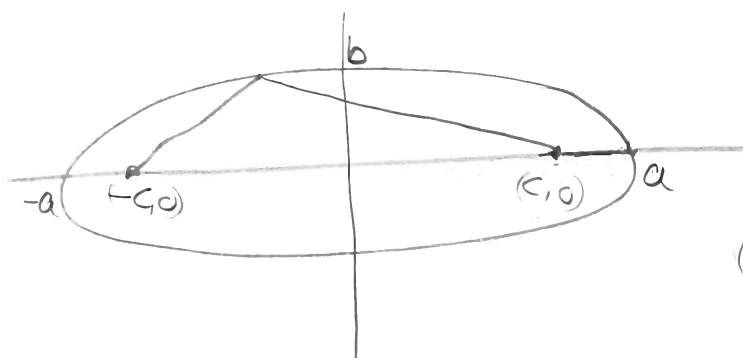
$$y^2 = 4cx$$

Ellipse: $\sqrt{(x-c)^2 + y^2} + \sqrt{(x+c)^2 + y^2} = 2a$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \underline{\underline{a^2 + b^2 = c^2}}$$

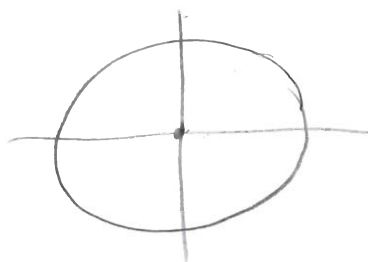
- Sentrum: $(0,0)$
- halvaksler $x=a$, $y=b$
- brennpunkter $(\pm c, 0)$

Exsentrisitet $e = \frac{c}{a}$ sier noe om hvor flatfylt ellipse er.



høy e (near 1)

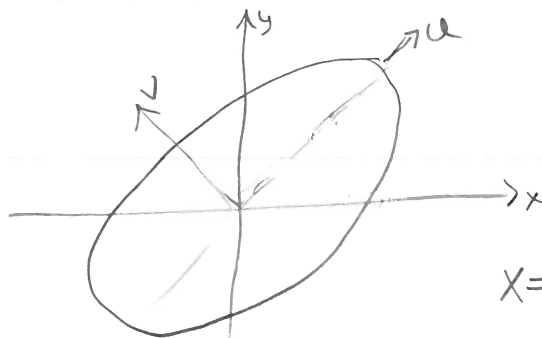
$$(a-c) + (a+c) = 2a$$



$e=0$ sirkel

Grensetilfelle $e=1$: $c=a$: En dobbelt linje

Hva med



$$\frac{u^2}{a^2} + \frac{v^2}{b^2} = 1$$

$$x = \frac{1}{\sqrt{2}}u - \frac{1}{\sqrt{2}}v \quad u = \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y$$

$$y = \frac{1}{\sqrt{2}}u + \frac{1}{\sqrt{2}}v \quad v = -\frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y$$

$$\frac{1}{a^2} \left(\frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y \right)^2 + \frac{1}{b^2} \left(-\frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y \right)^2 = 1$$

$$\frac{1}{a^2} \left(\frac{1}{2}x^2 + xy + \frac{1}{2}y^2 \right) + \frac{1}{b^2} \left(\frac{1}{2}x^2 - xy + \frac{1}{2}y^2 \right) = 1$$

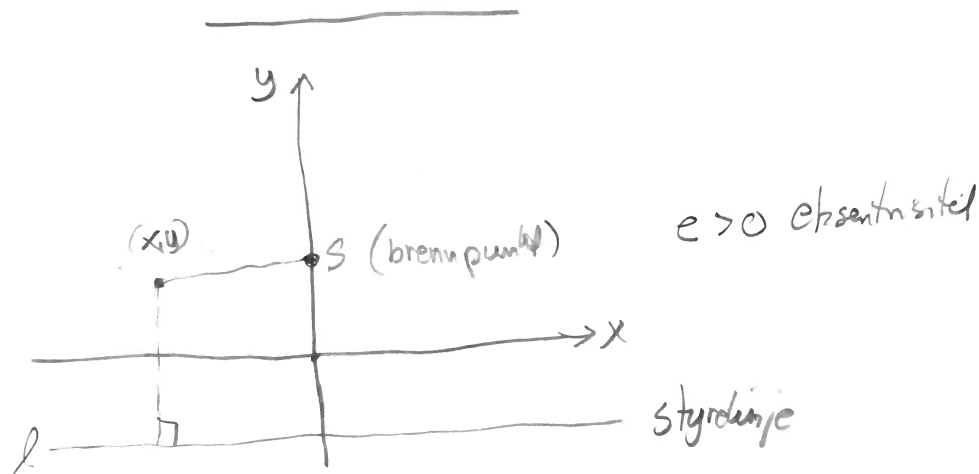
$$\frac{a^2+b^2}{a^2b^2} \cdot \frac{1}{2} x^2 + \frac{b^2-a^2}{a^2b^2} xy + \frac{a^2+b^2}{a^2b^2} \cdot \frac{1}{2} y^2 = 1$$

Exs $a=1, b=2$.

$$\frac{5}{4} \frac{1}{2} x^2 + \frac{4-1}{4} xy + \frac{5}{4} \frac{1}{2} y^2 = 1$$

$$\text{dvs } \frac{5}{8} x^2 + \frac{3}{4} xy + \frac{5}{8} y^2 = 1$$

$$\text{elle } \underline{5x^2 + 6xy + 5y^2 = 8}$$



geometrisk skal hvor $\frac{\text{afstand til } S}{\text{afstand til } l} = e$

$0 < e < 1$: ellipse

$e = 1$: parabel

$e > 1$: hyperbel

La $l: y = -1, S = (0, 1)$:

$$\frac{\sqrt{x^2 + (y-1)^2}}{y+1} = e \Leftrightarrow x^2 + y^2 - 2y + 1 = e^2 y^2 + 2e^2 y + e^2$$

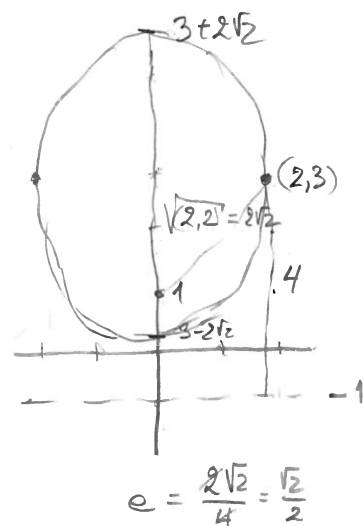
$$\Leftrightarrow x^2 + (1-e^2)y^2 - (2+2e^2)y = e^2 - 1$$

For $e=1$ $x^2 - 4y = 0$ $y = \frac{1}{4}x^2$

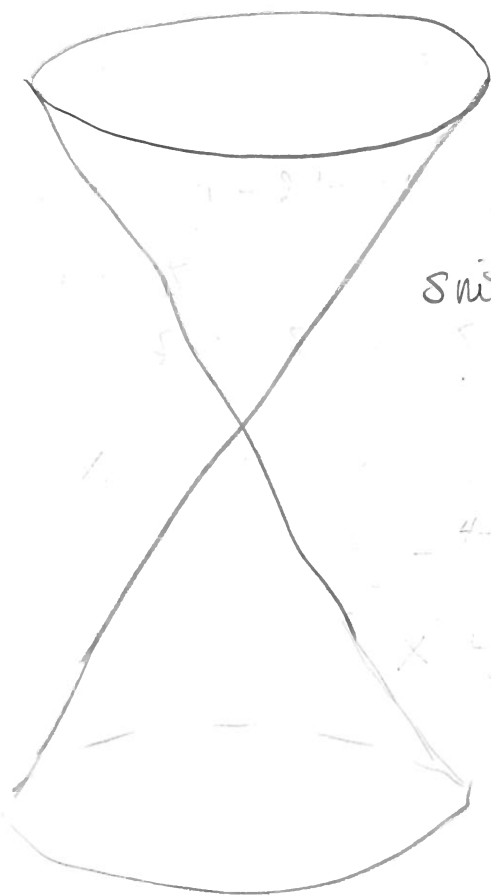
$$0 < e < 1 \quad \frac{x^2}{1-e^2} + \left(y - \frac{1+e^2}{1-e^2}\right)^2 = \frac{4e^2}{(1-e^2)^2}$$

$$e = \frac{\sqrt{2}}{2} \Rightarrow e^2 = \frac{1}{2} \quad 2x^2 + (y-3)^2 = 8$$

$$\text{dvs} \quad \frac{x^2}{2^2} + \frac{(y-3)^2}{(2\sqrt{2})^2} = 1$$



Keglesnitt:



$$z = x^2 + y^2$$

Snitt med planet $z = ax + b$