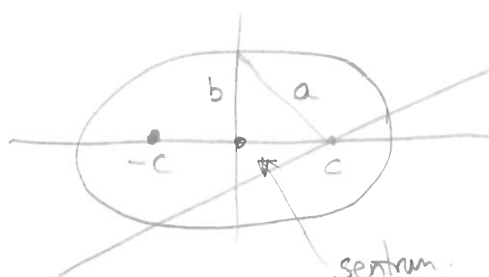


Mandag 14. september

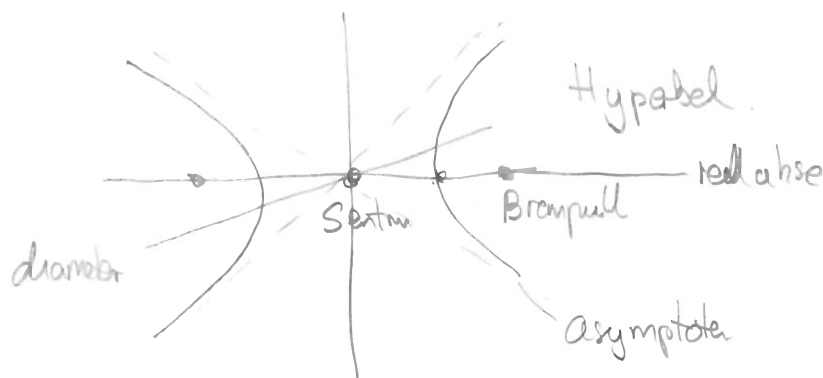
1. Keplers lover for planet bevegelse
(ligger på semesteriden)

Ellipse



skant 2 korde

store / lille halvakse
Brempunkter



$$x^2 - y^2 = 1$$

$$\left(\frac{x^2}{a^2} - \frac{y^2}{b^2} = \pm 1 \right)$$

Linjer i planet : $ax + by = c$ ($a, b \neq 0, 0$)
 $tax + tby = tc$ sammelinje

To linjer (ikke parallell) $ax + by = c$, $a'x + b'y = c'$

skjæringspunkt

$$\begin{array}{rcl} ax + by & = & c \quad | \cdot b' \quad a' \\ a'x + b'y & = & c' \quad | \cdot a \quad -a' \end{array}$$

$$(ab' - a'b)x = cb' - c'b$$

$$x = \frac{cb' - c'b}{ab' - a'b} \quad y = \frac{ca' - c'a}{ba' - b'a}$$

stall : $-\frac{a}{b} \alpha - \frac{a'}{b'}$

$$\begin{aligned} \tan(\alpha - \beta) &= \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)} = \frac{\sin \alpha \cos \beta - \cos \alpha \sin \beta}{\cos \alpha \cos \beta + \sin \alpha \sin \beta} = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \\ &= \frac{-\frac{a}{b} + \frac{a'}{b'}}{1 + \frac{aa'}{bb'}} = \frac{a'b - a'p}{a'a + p'b} \end{aligned}$$

Linjene står normalt på hverandre: $\alpha - \beta = \frac{\pi}{2}$, $\tan \frac{\pi}{2} = \infty$
 dvs. $aa' + bb' = 0$.

Ellipse: $\sqrt{(x-c)^2 + y^2} + \sqrt{(x+c)^2 + y^2} = 2a$

g. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $b^2 = \underline{\underline{a^2 - c^2}}$

Hyperbel $-\sqrt{(x-c)^2 + y^2} + \sqrt{(x+c)^2 + y^2} = 2a$

g. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $b^2 = \underline{\underline{a^2 - c^2}}$

Parabel $\sqrt{(x-c)^2 + y^2} = x + c$

g. $y^2 = 4cx$

Generelt. kjeglesnitt

$$ax^2 + bxy + cy^2 + ex + fy + g = 0$$

Bestemt av 5 parametre.

styres av $(x \ y) \begin{pmatrix} a & \frac{b}{2} \\ \frac{b}{2} & c \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

Opp til basissskifte: $(x' \ y') \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$

$$\lambda^2 - (a+c)\lambda + (ac - \frac{b^2}{4}) = 0$$

$$\Rightarrow \lambda = \frac{a+c \pm \sqrt{(a+c)^2 - 4(ac - \frac{b^2}{4})}}{2} = \frac{a+c}{2} \pm \frac{\sqrt{(a-c)^2 + b^2}}{2}$$

$$(a+c)^2 < (a-c)^2 + b^2 \quad \text{hyperbel}$$

$$(a+c)^2 > (a-c)^2 + b^2 \quad \text{ellipse}$$

$$a^2 + 2ac + c^2 > a^2 - 2ac + c^2 + b^2 \quad \text{dvs} \quad \begin{matrix} b^2 - 4ac < 0 & \text{ellipse} \\ > 0 & \text{hyperbel} \end{matrix}$$

Parametrisering.

Ellipse $x = a \cos \theta$ $y = b \sin \theta$

$$\text{eller } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Hyperbel $x = \frac{a}{\cosh t}$ $y = b \tanh t$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{1}{\cosh^2 t} - \tanh^2 t = \frac{1 - \sinh^2 t}{\cosh^2 t} = \underline{\underline{1}}$$

alternativt $x = a \cosh t$ $y = b \sinh t$

Tangent til $y = f(x)$ i $x = p$:

Linje gjennom $(p, f(p))$ med st. tall $f'(p)$

For ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Implisitt derivasjon mhp x : $\frac{2x}{a^2} + 2y \frac{dy}{dx} \frac{1}{b^2} = 0$

punkt (p, q) tilfredsstille $\frac{p^2}{a^2} + \frac{q^2}{b^2} = 1$

og $\frac{2p}{a^2} + 2q \frac{dy}{dx} \frac{1}{b^2} = 0$ dvs $\frac{dy}{dx} = -\frac{2pb^2}{a^2 2q} = -\frac{pb^2}{qa^2}$

Tangent: $y - q = -\frac{pb^2}{qa^2}(x - p) \quad \times \quad | \quad qa^2$

$$qa^2 y - q^2 a^2 = -pb^2 x + p^2 b^2 \quad | \cdot \frac{1}{a^2 b^2}$$

$$\frac{qy}{b^2} - \frac{q^2}{b^2} = -\frac{px}{a^2} + \frac{p^2}{a^2}$$

eller $\frac{px}{a^2} + \frac{qy}{b^2} = \frac{p^2}{a^2} + \frac{q^2}{b^2} = 1$

F. eks. A. $\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$ Tangent i punktet $(1, \frac{4\sqrt{2}}{3})$

$$\frac{1}{9} + \frac{32}{9 \cdot 4} = \frac{1+8}{9} = 1.$$

$p=1, q=\frac{4\sqrt{2}}{3} \quad a=3 \quad b=2$

$$\frac{1 \cdot x}{9} + \frac{\frac{4\sqrt{2}}{3} y}{4} = \frac{x}{9} + \frac{\sqrt{2} y}{3} = 1.$$

eller $x + 3\sqrt{2}y = 9$

$$1 + \frac{3\sqrt{2} \cdot 4\sqrt{2}}{3} = 1 + 8 = 9$$

B. $x^2 - y^2 = 1$ Punkt (x_0, y_0)

Tangent $x_0 x - y_0 y = 1$

$y_0 = 0 \quad x_0 = 1$ Tangent $x = 1$

$x_0 \rightarrow \infty \quad y_0 = \sqrt{x_0^2 - 1}$ Tangent $x_0 x - \sqrt{x_0^2 - 1} y = 1$

$$x_0 \left(x - \sqrt{1 - \frac{1}{x_0^2}} y \right) = 1$$

$x_0 \rightarrow \infty$ $x \pm y = 0$