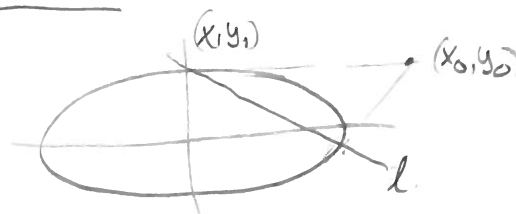


Torsdag 17. september

1

Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



Punkt (x_0, y_0) utenfor ellipse. Linja $l: \frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = 1$ kalles polaren til (x_0, y_0) (som er polen til linja).

Polaren skærer ellipsen i (x_1, y_1) og (x_2, y_2)

dvs $\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} = 1$, $\frac{x_0 x_1}{a^2} + \frac{y_0 y_1}{b^2} = 1$

$$\Rightarrow x_1 \frac{(x_1 - x_0)}{a^2} + y_1 \frac{(y_1 - y_0)}{b^2} = 0$$

$$\Rightarrow y_1 - y_0 = - \frac{x_1 b^2}{y_1 a^2} (x_1 - x_0)$$

Linja gjennom (x_1, y_1) og (x_0, y_0) er gitt ved

$$y - y_0 = \frac{y_1 - y_0}{x_1 - x_0} (x - x_0)$$

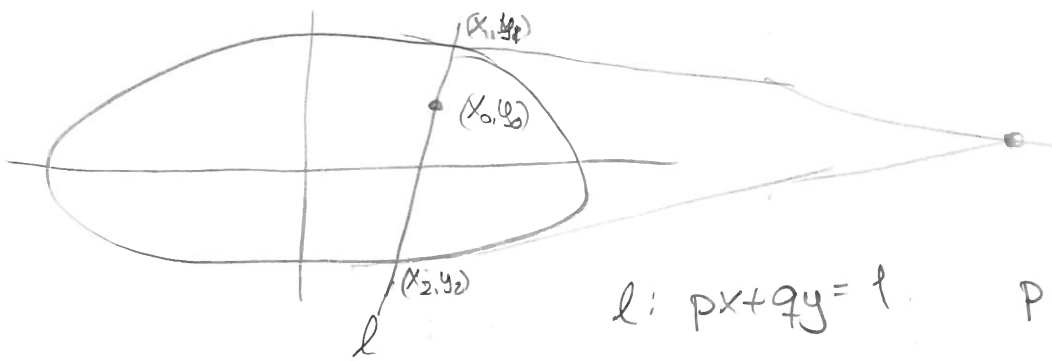
$$\begin{aligned} \Rightarrow y - y_0 &= \frac{x_1 b^2 (x_1 - x_0)}{y_1 a^2 (x_1 - x_0)} (x - x_0) \\ &= - \frac{x_1 b^2}{y_1 a^2} (x - x_0) \quad \left| \frac{y_1}{b^2} \right. \end{aligned}$$

$$\Rightarrow \frac{y_1 y}{b^2} - \frac{y_0 y_1}{b^2} = \frac{x_1 x_0}{a^2} - \frac{x_1 x}{a^2}$$

$$\text{eller } \frac{y_1 y}{b^2} + \frac{x_1 x}{a^2} = \frac{x_1 x_0}{a^2} + \frac{y_0 y_1}{b^2} = 1$$

Linjene fra polen til skjæringspunktene mellom ellipsen og polaren er tangenter til ellipsen.

Korollar: Pol $\in$ polare $\Leftrightarrow$ pol $\in$ ellipsen
polaren er tangent



$$l: px + qy = 1 \quad px_i + qy_i = 1 \quad i=0,1,2$$

Tangenter i (x_j, y_j) : $\frac{x x_j}{a^2} + \frac{y y_j}{b^2} = 1 \quad j=1,2$

Tre punkter på l : $\begin{vmatrix} x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = x_0(y_1 - y_2) + y_0(x_2 - x_1) + x_1 y_2 - x_2 y_1 = 0$ *

skjæring mellom tangentene

$$\frac{x_1}{a^2} x + \frac{y_1}{b^2} y = 1$$

$$\frac{x_2}{a^2} x + \frac{y_2}{b^2} y = 1$$

har løsning $(x, y) = \left(\frac{a^2(y_1 - y_2)}{y_1 x_2 - y_2 x_1}, \frac{b^2(x_2 - x_1)}{y_1 x_2 - y_2 x_1} \right)$

Dette punktet ligger på linja $\frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = 1$ (polaren til (x_0, y_0))

ford.

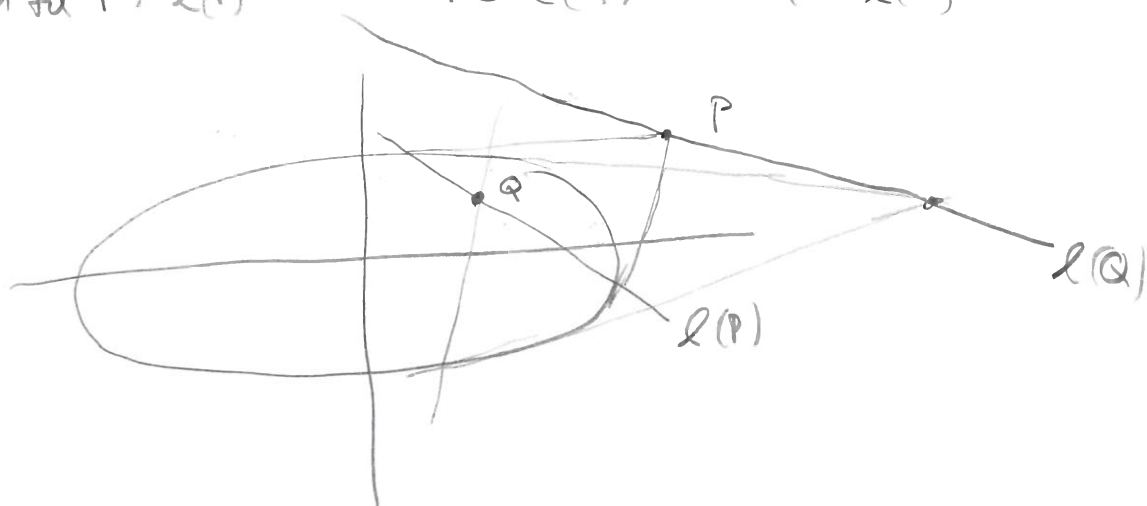
$$\frac{a^2 x_0 (y_1 - y_2)}{a^2 (y_1 x_2 - y_2 x_1)} + \frac{b^2 y_0 (x_2 - x_1)}{b^2 (y_1 x_2 - y_2 x_1)}$$

$$= \frac{x_0 (y_1 - y_2) + y_0 (x_2 - x_1)}{y_1 x_2 - y_2 x_1} = 1 \quad \text{pga } *$$

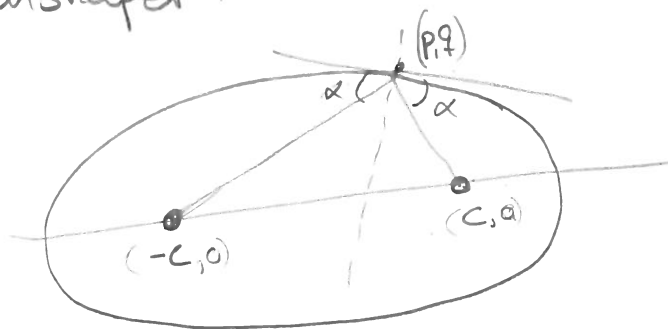
La Hire's theorem:

polaren til P : $l(P)$

$$P \in l(Q) \Leftrightarrow Q \in l(P)$$



Spiegelungsegenskaper.



Tangent i (p, q) : $\frac{x p}{a^2} + \frac{y q}{b^2} = 1$

Linje til brennpunkter $y = \frac{q}{p \pm c}(x \mp c) = \frac{q}{p \pm c}(x \pm c)$
 $(p \pm c)y - qx = \pm c$

$$\begin{aligned} Ax + By &= C \\ A'x + B'y &= C' \\ \tan(\alpha - \beta) &= \frac{-AB' + A'B}{AA' + BB'} \end{aligned}$$

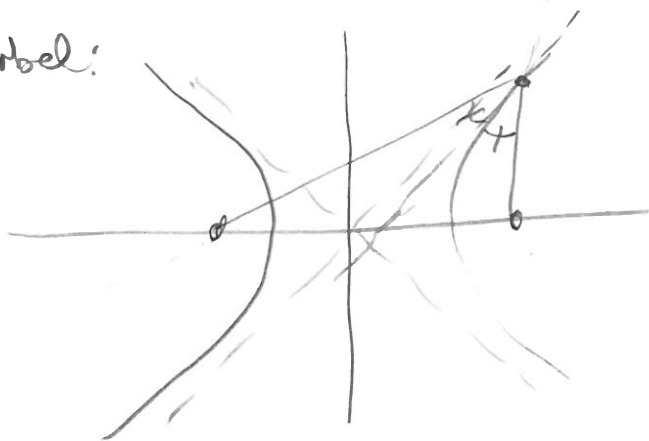
$$A = \frac{x}{a^2} \quad B = \frac{y}{b^2}$$

$$A' = -q \quad B' = p \pm c \quad \text{eller} \quad A' = -q \quad B' = p - c$$

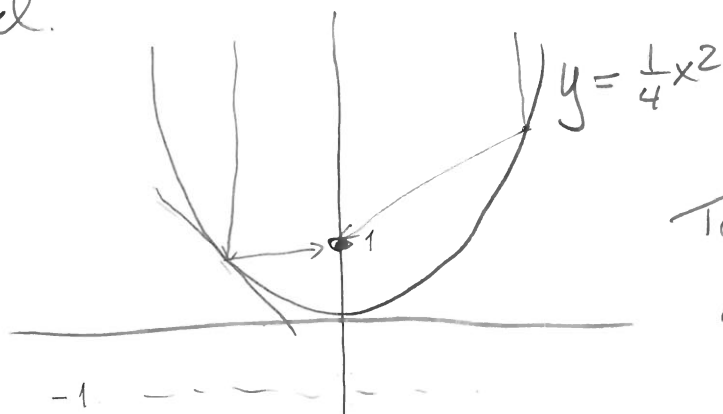
$\Downarrow$

$$\begin{aligned} \tan \alpha &= \frac{-\frac{p}{a^2}(p \pm c) - q \frac{q}{b^2}}{-\frac{p}{a^2}q + \frac{q}{b^2}(p \pm c)} = \frac{\mp \frac{pc}{a^2} - 1}{p q \left(\frac{1}{b^2} - \frac{1}{a^2} \right) \pm \frac{qc}{b^2}} \cdot \frac{a^2 b^2}{a^2 b^2} \\ &= \mp \frac{a^2 b^2 \pm pc b^2}{p q c^2 \pm q c a^2} = - \frac{b^2 (a^2 \pm pc)}{q c (pc \pm a^2)} = \begin{cases} -\frac{b^2}{qc} \\ \frac{b^2}{qc} \end{cases} \end{aligned}$$

Hyperbel:



Parabel:



$$\sqrt{x^2 + (y-1)^2} = y+1$$

$$\Downarrow$$

$$x^2 + y^2 - 2y + 1 = y^2 + 2y + 1$$

$$\Downarrow$$

$$y = \frac{1}{4}x^2$$

Tangent: st. tall $\frac{dy}{dx} = \frac{1}{2}x$ i punkt $(p, \frac{1}{4}p^2)$

$$y - \frac{1}{4}p^2 = \frac{1}{2}p(x - p)$$

$$\Updownarrow$$

$$2px - 4y = p^2$$

Vinkel med vertikal: $x + 0y = p$

$$\text{tg} = \frac{4}{2p} = \frac{2}{p}$$

Vinkel med $y-1 = \frac{\frac{1}{4}p^2 - 1}{p}(x - p)$ 1.4p

$$\text{dvs } (p^2 - 4)x - 4py = *$$

$$\text{tg} = \frac{-(p^2 - 4)(-4) + (-4p) \cdot 2p}{(p^2 - 4)2p + (-4)(-4p)}$$

$$= \frac{4p^2 - 16 - 8p^2}{2p^3 - 8p + 16p} = \frac{-4p^2 - 16}{2p^3 + 8p} = -\frac{2}{p}$$

(x,y)

 (p_1, q_1) (p_2, q_2)

$$\sqrt{(x-p_1)^2 + (y-q_1)^2} - \sqrt{(x-p_2)^2 + (y-q_2)^2} = d$$

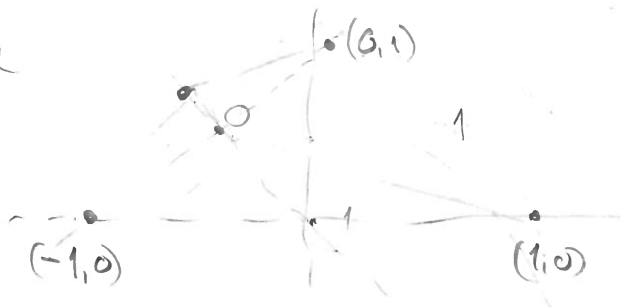
Hyperbel med brennpunkter (p_1, q_1) (p_2, q_2)

Tre punkter:

To punkter: En hyperbel

Tre punkter: Ett punkt.

Eksempel



$$\sqrt{x^2 + (y-1)^2} - \sqrt{(x-1)^2 + y^2} = 1$$

$$\sqrt{(x+1)^2 + y^2} - \sqrt{(x-1)^2 + y^2} = 1$$

$$\sqrt{x^2 + (y-1)^2} - \sqrt{(x+1)^2 + y^2} = 0 \Rightarrow x = -y$$

$$y = \frac{\sqrt{6}}{4}$$