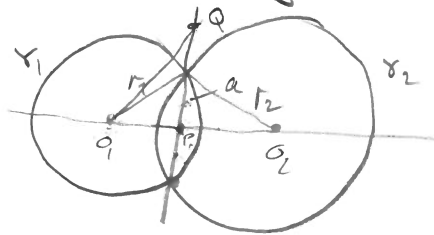


1.

Radikal^{gæsen}~~te~~ : Alle punkter som har samme potens med hensyn på to siddler

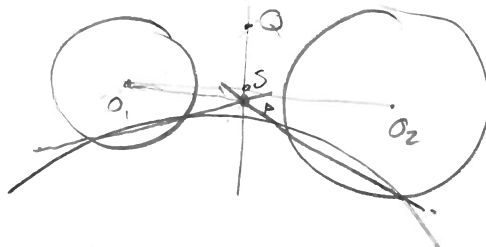


$$(r_1 - OP_1)(r_1 + OP_1) = r_1^2 - OP_1^2 = r_1^2 - r_1^2 + a^2 = a^2$$

$$O, Q = \sqrt{O, P^2 + P, Q^2}$$

Potens: $(O_1Q - r_1)(O_1Q + r_1) = O_1Q^2 - r_1^2$

$$\begin{aligned} &= O_1P^2 + P_1Q^2 - r^2 \\ &= PQ^2 + O_1P^2 - r^2 \\ &= \underline{PQ^2 - a^2} \end{aligned}$$



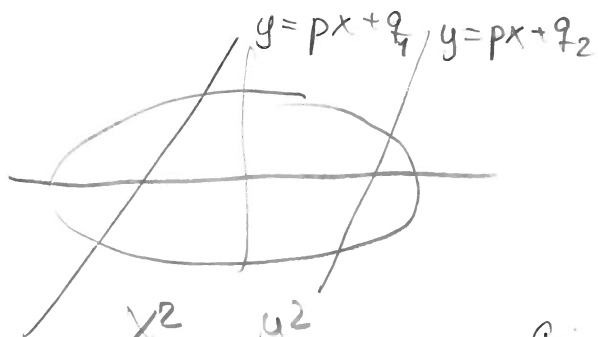
$$O_1 S^2 - P S^2 - r_1^2 = O_2 S^2 - P S^2 - r_2^2$$

$$\Rightarrow$$

$$O_1 P^2 - r_1^2 = O_2 P^2 - r_2^2$$

$$\Rightarrow S \in \text{radical axis}$$

$$\begin{aligned}(O_1Q - r_1)(O_1Q + r_1) &= O_1Q^2 - r_1^2 \\ &= O_1S^2 + SO^2 - r_1^2 = O_2S^2 + SO^2 - r_2^2 \\ &= O_2Q^2 - r_2^2\end{aligned}$$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Skjæringspunkt

$$\frac{x^2}{a^2} + \frac{(px+q_i)^2}{b^2} = 1$$

$$\text{gi } (b^2 + a^2 p^2) x^2 + 2a^2 p q_i x + a^2 (q_i^2 - b^2) = 0$$

Middelpunkt

$$x = \frac{-2a^2 p q_i}{2(b^2 + a^2 p^2)} \quad y = \frac{-a^2 p^2 q_i}{b^2 + a^2 p^2} + q_i$$

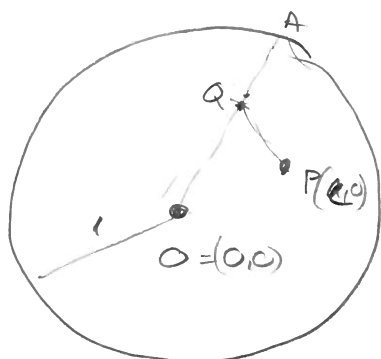
Linje gjennom de to punktene og origo:

$$b^2 + a^2 p^2 = N \quad \begin{vmatrix} \frac{-a^2 p q_1}{b^2 + a^2 p^2} & \frac{-a^2 p^2 q_1}{b^2 + a^2 p^2} + q_1 & 1 \\ 0 & 0 & 1 \\ \frac{-a^2 p q_2}{b^2 + a^2 p^2} & \frac{-a^2 p^2 q_2}{b^2 + a^2 p^2} + q_2 & 1 \end{vmatrix}$$

$$= \left(\frac{-a^2 p^2 q_1}{N} + q_1 \right) \left(-\frac{a^2 p q_2}{N} \right) - \left(\frac{-a^2 p^2 q_2}{N} + q_2 \right) \left(-\frac{a^2 p q_1}{N} \right)$$

$$= \frac{1}{N^2} \left(a^4 p^3 q_1 q_2 - a^2 p q_1 q_2 N - a^4 p^3 q_1 q_2 + a^2 p q_1 q_2 N \right)$$

$$= 0$$



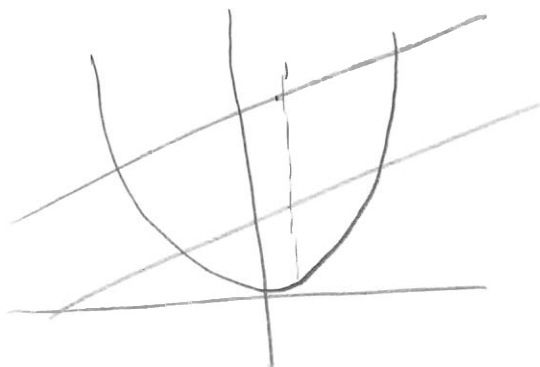
$$\underline{\underline{QA = QP}}$$

$$Q = (x, y) \Rightarrow QA = 1 - \sqrt{x^2 + y^2}$$

$$QP = \sqrt{(x-c)^2 + y^2}$$

$$\Rightarrow \sqrt{x^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 1$$

ellipse



5.
 Ges. sted for midtpunkt på
 parallelle kordi ligger på linje
 som er parallel med symmetriaksen

$$y = x^2$$

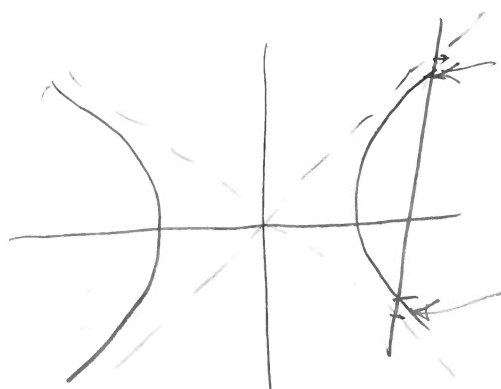
$$y = px + q$$

$$\Rightarrow (px + q) = x^2$$

$$\text{dvs } x^2 - px - q = 0$$

$$\Rightarrow x = \frac{p \pm \sqrt{p^2 + 4q}}{2}$$

$$\text{Midtpunkt } x = \frac{p}{2} \quad y = \frac{p^2}{4} \quad \text{uafhængig af } q.$$



hvis store

Ans. 5.3

$$t \in \mathbb{R} \quad (1+t)x^2 + (1-t)y^2 = 36$$

Ellipse/Hyperbel $1+t, 1-t > 0$



Tangent med st. tall 1 og ges. sted til tangentens punkt

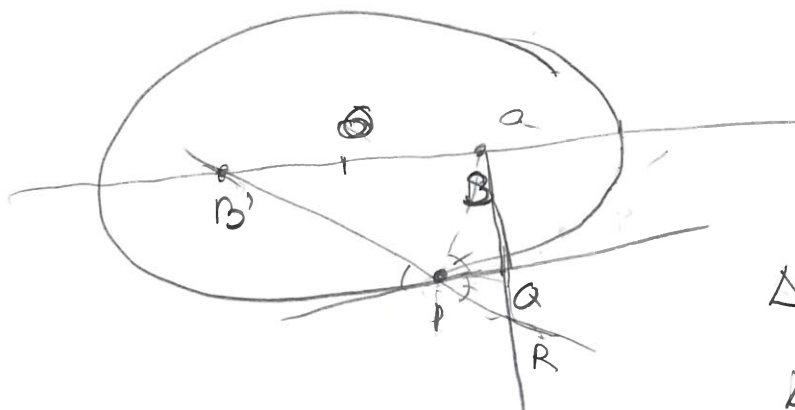
$$P = (x_0, y_0) \text{ på } C \quad \text{Tangent } (1+t)x_0x + (1-t)y_0y = 36$$

$$\text{st. tall} = 1 \quad \text{betyr } (1+t)x_0 = -(1-t)y_0$$

$$\left. \begin{array}{l} \text{Tangentens punkt} \\ (1+t)x_0 + (1-t)y_0 = 0 \\ (1+t)x_0^2 + (1-t)y_0^2 = 36 \end{array} \right\} \begin{array}{l} 2x_0y_0 = 36 \\ \text{dvs hyperbel} \end{array}$$

Pedal kurve.

Geo. sted for fotpunktet til
en normal fra S på en tangent til kurven



$\triangle PBQ$ - PQ felles

$\triangle PQR$ - $BQ = RQ$

$\Rightarrow$ Kongruente

$\Rightarrow PR = PB, B'R = 2a$

$OB = OB', QB = QR$

$\Rightarrow BQ = \frac{1}{2} B'R = a \Rightarrow$ sirkel med radius a .

Storsirkelen til ellipsen

H17 oppg 4

$C: x^2 + y^2 = 1 \quad A = (2, 0)$

$S \in C$: Geo. sted for midtpunktet P mellom S og A.

- Finn likning

- Finn likning til tangenten til C gjennom A