

Mandag 5. oktober

1

Symmetrier av polyedre

Tetrueder



$$S_1 = S_{\frac{\pi}{2}}$$

rotasjon om et hjørne (234)

$$S_2$$

rotasjon om et annet hjørne (123)

12 symmetrier

$$S_1 S_2 = (13)(24)$$

$$S_2 S_1 = (12)(34)$$

$$S_1 S_2 S_1^2 = (134) = S_2^2 S_1$$

$$S_1^2 = (243)$$

$$S_2^2 = (132)$$

$$S_1 S_2^2 = (234)(132) = (142)$$

$$S_2^2 = S_1 S_2 S_1 = (132)$$

$$S_2 S_1 S_2 = (243) = S_1^2$$

$$S_2 S_1^2 = (123)(243) = (124)$$

Symmetrier: $\{ \text{id}, (12)(34), (13)(24), (14)(23), (123), (132), (124), (142), (134), (143), (234), (243) \}$

$$= \{ \text{id}, S_2 S_1, S_1 S_2, S_2 S_1^2 S_2, S_2, S_2^2, S_2 S_1^2, S_1 S_2^2, S_2^2 S_1, S_1^2 S_2, S_1, S_1^2 \}$$

$$S_1^3 = S_2^3 = \text{id} \quad S_2 S_1 S_2 S_1 = \text{id}$$

$$S = S_1 = (123)$$

$$S^3 = \text{id}$$

$$(st)^3 = \text{id}$$

$$t = S_2 S_1 = (12)(34)$$

$$t^2 = \text{id}$$

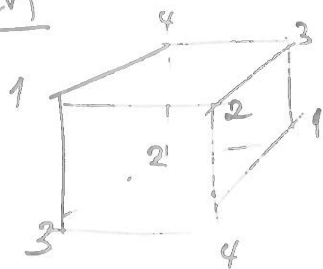
$$S = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$t = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

A₄

12 elementer

$$\{ \underline{S = S_2}, \underline{S^2 t = S_1} \}$$

Kuben

24 symmetrier.

$$\begin{array}{l} \sigma: (1234) \\ \mu: (1342) \end{array} \quad \left. \vphantom{\begin{array}{l} \sigma \\ \mu \end{array}} \right\} \text{orden 4.}$$

$$\sigma\mu: \begin{array}{ccccc} 1 & \rightarrow & 3 & \rightarrow & 4 \\ 2 & & 1 & & 2 \\ 3 & & 4 & & 1 \\ 4 & & 2 & & 3 \end{array} \cdot (143) \quad \text{orden 3}$$

$\mu \quad \sigma$

$$\text{orden 2 : } (1234)(143) : \begin{array}{ccccc} 1 & & 3 & & 4 \\ 2 & \rightarrow & 1 & \rightarrow & 2 \\ 3 & & 4 & & 1 \\ 4 & & 2 & & 3 \end{array} \cdot (23)$$

$\sigma^2\mu$ $\mu \quad \sigma \quad \sigma$

S_4 full permutasjonsgruppe
på 4 elementer.

Dodekaeder

12 5-kanter :: 60 symmetrier

 A_5 : alternerende gruppe på 5 elementer

Permutasjoner som er satt sammen
av transposisjoner (12)

$$(12345) = (12)(23)(34)(45)$$

Alternerende gruppe A_5 : Partall antall transposisjoner

Endelige undergrupper av $Isom_3$:



Hver rotasjon definerer to poler p

G_p : alle rotasjoner som fikserer p

endelig $\Rightarrow G_p \cong \mathbb{Z}_{r_p}$ (syklisk gruppe)

Banen til p : G_p er alle poler som G flytter rundt på.

$$|G| = N$$

$$|G| = |G_p| \cdot (\text{antall forskjellige poler som } G \text{ flytter rundt på})$$

$$r_p \cdot n_p = |G| \quad \forall p$$

$$\{(g, p) \mid \begin{array}{l} g: \text{symmetri} \\ \uparrow \\ \text{rotasjon} \end{array} \quad p: \text{pol for } g.\}$$

Antall symmetrier med p som pol (unntatt id)

$$r_p - 1$$

$$\sum_{p \text{ pol}} r_p - 1 = \sum_{i=1}^m n_i (r_i - 1)$$

$$i = 1, \dots, m$$

distinkte poler

$$n_i = |\text{banen } B_i|$$

Hver symmetri (unntatt id) har to poler

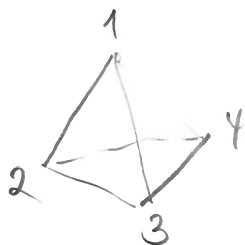
$$n_i r_i = N$$

selv om de er forskjellige

$$2(N-1)$$

$$\Rightarrow 2(N-1) = \sum n_i (r_i - 1)$$

$$\Rightarrow 2 - \frac{2}{N} = \sum_{i=1}^m 1 - \frac{1}{r_i}$$



12 symmetries:

$$12 \left\{ \begin{array}{l} \text{Id} \\ (123) \quad (132) \\ (124) \quad (142) \\ (134) \quad (143) \\ (234) \quad (243) \\ (12)(34) \\ (13)(24) \\ (14)(23) \end{array} \right.$$

Polar

$$\left. \begin{array}{l} - \\ 4, 123 \\ 3, 124 \\ 2, 134 \\ 1, 234 \end{array} \right\} \text{stabilisator } \mathbb{Z}_3$$

$$\left. \begin{array}{l} 12, 34 \\ 13, 24 \\ 14, 23 \end{array} \right\} \text{stabilisator } \mathbb{Z}_2$$

$$\text{Banan: } \{1, 2, 3, 4\} \quad \{123, 124, 134, 234\} \quad \{12, 13, 14, 23, 24, 34\}$$

$$4 \cdot 3 = 12 \quad 4 \cdot 3 = 12 \quad 6 \cdot 2 = 12$$

$$\{(g, p) \mid p \text{ pol for } g\}$$

$$\begin{array}{llll} (id, 1) & (id, 123) & (id, 12) & ((123), 4) \quad (123), 123) \quad (132), 4) \quad (132), 123) \\ (id, 2) & (id, 124) & (id, 13) & (124, 3) \\ (id, 3) & (id, 134) & (id, 14) & (134, 2) \\ (id, 4) & (id, 234) & (id, 23) & (234, 1) \\ & & (id, 24) & \\ & & (id, 34) & (12)(34), 12) \quad (12)(34), 34) \\ & & & (13)(24), 13) \quad - \quad - \\ & & & (14)(23), 14) \quad - \quad - \end{array}$$

$$N=12$$

$$2(N-1) = 2 \cdot 11 = 22$$

$$\sum n_i(r_i-1) = 4(3-1) + 4(3-1) + 6(2-1) = 22$$

$$2 - \frac{2}{12} = 1 - \frac{1}{3} + 1 - \frac{1}{3} + 1 - \frac{1}{2}$$

$$\parallel$$

$$\frac{11}{6}$$

$$\frac{2}{3} + \frac{2}{3} + \frac{1}{2} = \frac{4+4+3}{6} = \frac{11}{6}$$

$$2 - \frac{2}{N} = \sum_{i=1}^m 1 - \frac{1}{r_i}$$

$$\underline{m=1}$$

$$2 - \frac{2}{N} = 1 - \frac{1}{r} < 1.$$

$$N \geq 2 \Rightarrow \frac{2}{N} \leq 1 \Rightarrow 2 - \frac{2}{N} \geq 2 - 1 = 1.$$

umulig

$$\underline{m=2}$$

$$2 - \frac{2}{N} = 2 - \frac{1}{r_1} - \frac{1}{r_2}$$

$$\Rightarrow \frac{2}{N} = \frac{1}{r_1} + \frac{1}{r_2}$$

Ma'ha $r_i \leq N$

$$\text{Hvis } r_i < N, \text{ s\aa } \frac{1}{r_i} > \frac{1}{N} \text{ s\aa } \frac{1}{r_1} + \frac{1}{r_2} > \frac{2}{N}$$

$$\Rightarrow \text{dvs } r_1 = r_2 = N$$

$$\Rightarrow n_1 = n_2 = 1$$

Kun en pol.

$$\underline{m=3}$$

$$2 - \frac{2}{N} = 3 - \frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3}$$

Kan anta

$$r_1 \leq r_2 \leq r_3$$

$$\Rightarrow \frac{2}{N} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} - 1$$

$$\underline{r_1=2}$$

$$\frac{2}{N} = \frac{1}{r_2} + \frac{1}{r_3} - \frac{1}{2}$$

$$\text{Hvis } r_2=2, \frac{2}{N} = \frac{1}{r_3}$$

$\Downarrow$

$$N = 2r_3$$

$\Rightarrow$ Sykkesh eller dekkesh

Hvis $r_2 \geq 3$

$$\frac{1}{r_2} + \frac{1}{r_3} - \frac{1}{2} \leq \frac{1}{3} + \frac{1}{r_3} - \frac{1}{2}$$

$$= \frac{1}{r_3} - \frac{1}{6}$$

 $r_3 \geq 6$ qui negativt

$$\Rightarrow r_3 = 3, 4, 5$$

Hvis $r_2 \geq 4 \Rightarrow r_3 \geq 4$.

$$\frac{1}{r_2} + \frac{1}{r_3} - \frac{1}{2} \leq 0 \quad \text{umulig.}$$

 $\Rightarrow$ 3 muligheder

$$(2, 3, 3) \quad (2, 3, 4) \quad (2, 3, 5)$$



Tetraeder

Kube
OktaederIkosaeder
Dodekaeder