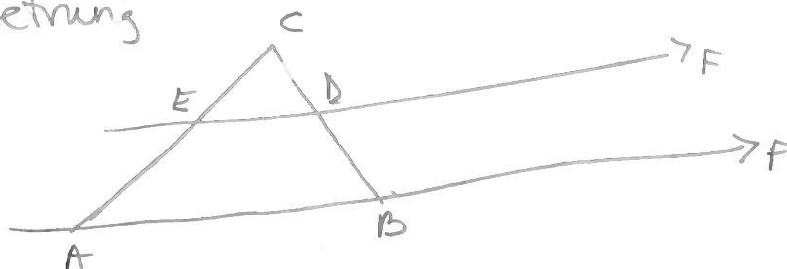


Mandag 12. oktober



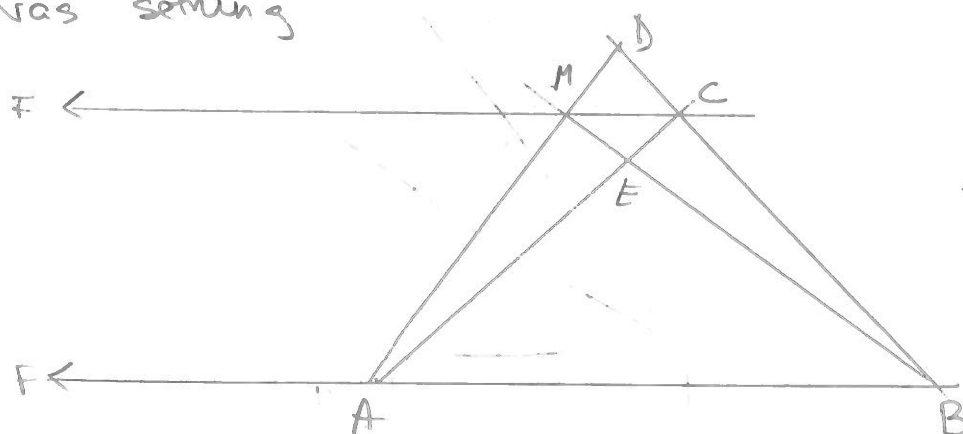
$$\frac{\overrightarrow{AP}}{\overrightarrow{PB}} \xrightarrow{P \rightarrow A} -1$$

Menelaos setning



$$\frac{BD}{DC} = \frac{AE}{EC} \Rightarrow \frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = \frac{AF}{FB} = -1$$

Cevas setning

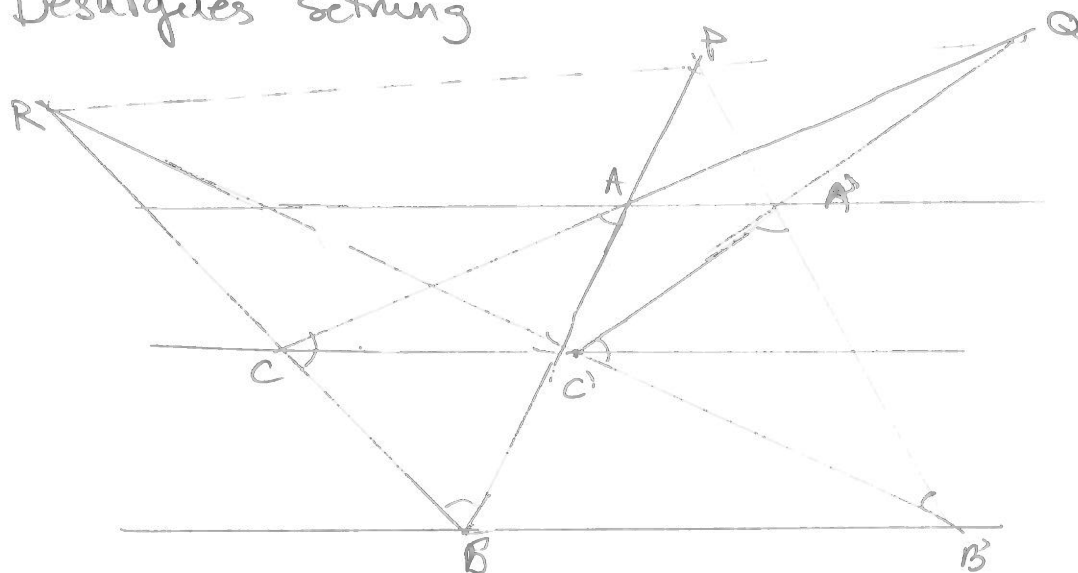


$$\begin{aligned} \triangle MDC &\sim \triangle ADB \\ \triangle AEB &\sim \triangle CEM \end{aligned}$$

$$\Rightarrow \frac{BD}{DC} = \frac{AB}{CM}, \quad \frac{CE}{EA} = \frac{CM}{AB}$$

$$\Rightarrow \frac{BD}{DC} \cdot \frac{CE}{EA} = \frac{AB}{CM} \cdot \frac{CM}{AB} = 1$$

## Desargues's Lemma



$$\frac{B'P}{PA} \cdot \frac{AQ}{QC} \cdot \frac{C'R}{RB} = \frac{BP}{PA} \cdot \frac{AQ}{QC} \cdot \frac{CR}{RB} = \frac{B-B'}{A-A'} \cdot \frac{A-A'}{C-C'} \cdot \frac{C-C'}{B-B'} = 1$$

$\Rightarrow R, P, Q$  collinear

Example Given to linear  $x_0 - 2x_1 + 3x_2 = 0$  and  $3x_0 + x_1 + 4x_2 = 0$   
 Syzygy point:

$$\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & -2 & 3 \\ 3 & 1 & 4 \end{vmatrix} = -11\bar{i} + 5\bar{j} + 7\bar{k}$$

Line given by  $(-11:5:7)$  or  $(0:1:0) \in \mathbb{P}^2$

$$\begin{vmatrix} x & y & z \\ -11 & 5 & 7 \\ 0 & 1 & 0 \end{vmatrix} = -7x - 11z \neq 0$$

# Projektive transformasjoner

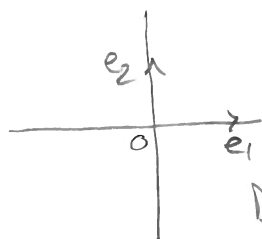
$$F: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

- bijektiv

- tar linjer på linjer

$$- F(0) = 0$$

} skal vise at  $F$  er en lineær-avbildning



$$F(e_1) = v_1 \quad F(e_2) = v_2$$

Definer  $G: \mathbb{R}^2 \rightarrow \mathbb{R}^2$   $G(v_1) = e_1 \quad G(v_2) = e_2$   
lineær avbildning

$$\text{Sett } H = G \circ F$$

- bijektiv

- tar linjer på linjer } parallelle linjer på parallell linje

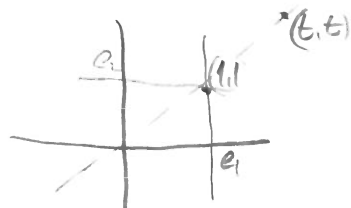
$$- H(0) = 0, \quad H(e_i) = e_i \quad i=1,2$$

Hva med  $H$  brukt på andre punkter?

$$H(se_1) = f_1(s) e_1$$

$$H(te_2) = f_2(t) e_2$$

$$f_i: \mathbb{R} \rightarrow \mathbb{R} \quad \begin{aligned} f_i(0) &= 0 \\ f_i(1) &= 1 \end{aligned}$$



$H(e_1 + te_2)$  må avbildes på seg selv  
fordi  $H(e_1) = e_1$  og  $e_1 + te_2$  er  
parallelle med  $y$ -aksen

Tilsvarende for  $H(se_1 + e_2)$

$$\Rightarrow H(1,1) = (1,1) \quad \text{skyenningen mellom de to linjene}$$

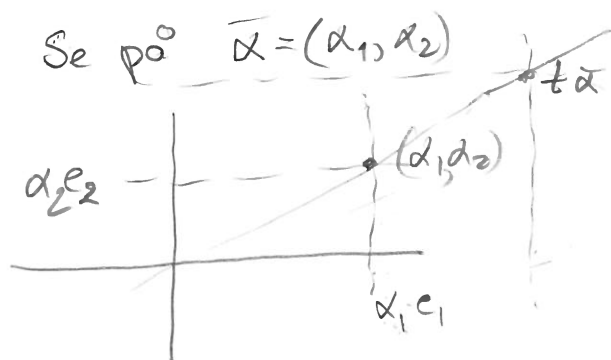
$$H(t,t) = (f_1(t), f_2(t)) \quad \text{sidan } (t,t) \in te_1 + \mathbb{R}e_2 \mapsto f_1(t)e_1 + \mathbb{R}e_2$$

$$= (f_2(t), f_2(t))$$

Samme argument.

$$\Rightarrow f_1 = f_2$$

$$\Rightarrow H(se_i) = f(s) e_i \quad i=1,2.$$



$$H(t\vec{\alpha}) = f(t\alpha_1)e_1 + f(t\alpha_2)e_2$$

$$= S_{\alpha}(t) H(\vec{\alpha})$$

$$= S_{\alpha}(t) f(\alpha_1)e_1 + S_{\alpha}(t) f(\alpha_2)e_2$$

$$\vec{\beta} = (\beta_1, \beta_2) : H(t\vec{\beta}) = S_{\beta}(t) f(\beta_1)e_1 + S_{\beta}(t) f(\beta_2)e_2$$

$t\vec{\alpha}$  og  $t\vec{\beta}$  ligger på linja  $t\alpha_1 e_1 + \mathbb{R}e_2$  og  $H(t\vec{\alpha})$  og  $H(t\vec{\beta})$  må ligge på samme linje, parallell med y-akser

$$\Rightarrow S_{\alpha}(t) = S_{\beta}(t) = S(t)$$

$$f(t\alpha_1) = S(t) f(\alpha_1) \quad \text{Sett } \alpha_1 = 1 : f(t) = S(t)$$

$$\alpha_1 = u \Rightarrow S(tu) = S(t) \cdot S(u)$$

multiplikativ

Et (langt og teknisk) argument viser at  $S$  også er additiv, d.e.

$$S(t+u) = S(t) + S(u)$$

Oppsummert

$$S(0) = 0$$

$$S(1) = 1$$

$$S(tu) = S(t) S(u)$$

$$S(t+u) = S(t) + S(u)$$

Additiv  $\Rightarrow$  Kontinuerlig

$$f(1)=1 \Rightarrow f\left(\frac{1}{x}\right) = \frac{1}{f(x)} \Rightarrow f \text{ identitet på } \mathbb{Q}$$

$$\Rightarrow \text{Identitet på } \mathbb{R}$$

$$\Rightarrow \underline{\underline{F \text{ er en lineær avbildning}}}$$

$$\mathbb{P}^2: (x_0: x_1: x_2)$$

$$\mathbb{R}^3 / \sim \quad (x_0, x_1, x_2) \sim (tx_0, tx_1, tx_2) \\ (\text{linjer gjennom origo})$$

Lineærafbildning:  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$

$$\bar{x} \sim t\bar{x} \Rightarrow T(t\bar{x}) = t \cdot T(\bar{x})$$

$$\Rightarrow T(\bar{x}) \sim T(t\bar{x})$$

alts.  $T: \mathbb{P}^2 \rightarrow \mathbb{P}^2$  projektiv transformasjon

Setning:

Gitt to <sup>ordrede</sup> sett av 4 punkter i generell posisjon i  $\mathbb{P}^2$ . Da finnes <sup>entydig</sup>  $T: \mathbb{P}^2 \rightarrow \mathbb{P}^2$  som tar den ene mengden på den andre

Beris: Betrakt mengden  $(1:0:0), (0:1:0), (0:0:1), (1:1:1)$

$$\text{og } P_i = (x_i: y_i: z_i) \quad i = 1, 2, 3, 4$$

$$\text{La } P_4 = \lambda_1 P_1 + \lambda_2 P_2 + \lambda_3 P_3 \text{ og sett}$$

$$M = \begin{pmatrix} \lambda_1 x_1 & \lambda_2 x_2 & \lambda_3 x_3 \\ \lambda_1 y_1 & \lambda_2 y_2 & \lambda_3 y_3 \\ \lambda_1 z_1 & \lambda_2 z_2 & \lambda_3 z_3 \end{pmatrix}$$

$$\Rightarrow M \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \lambda_1 \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \sim (x_1: y_1: z_1) = P_1$$

Tilsv. for  $P_2, P_3$

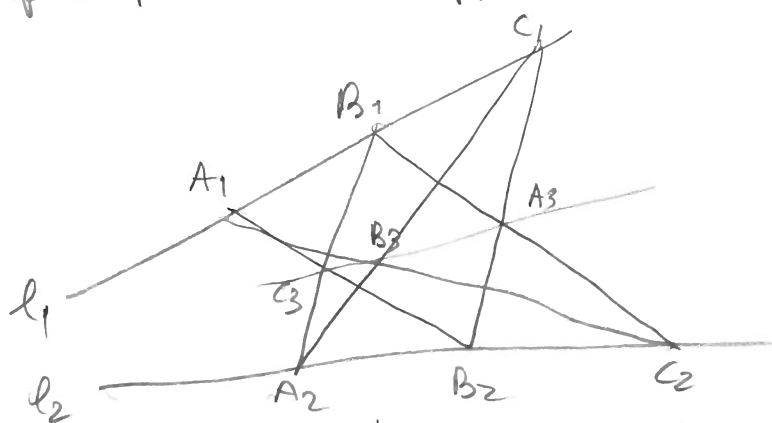
$$M \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \lambda_1 P_1 + \lambda_2 P_2 + \lambda_3 P_3 = P_4 \quad \square$$

Entydighet

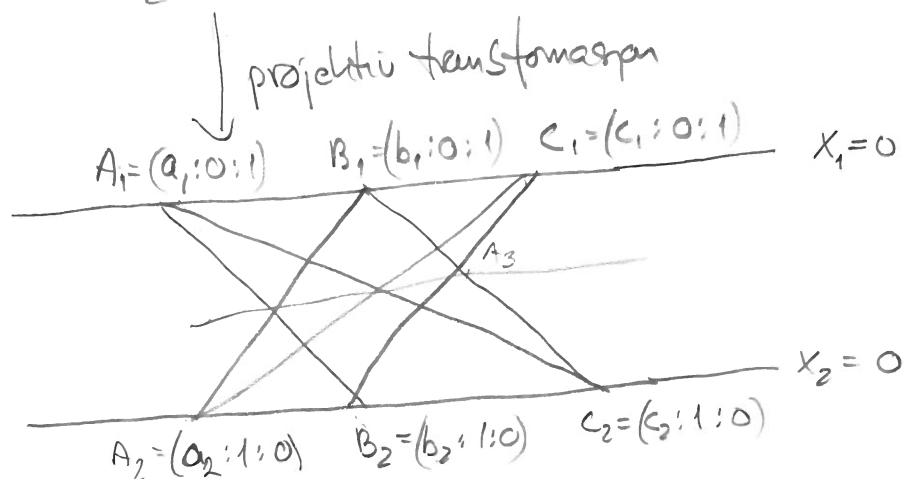
$$U: \begin{pmatrix} 1:0:0 \\ 0:1:0 \\ 0:0:1 \end{pmatrix} \mapsto \begin{pmatrix} 1:0:0 \\ 0:1:0 \\ 0:0:1 \end{pmatrix} \Rightarrow U = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

$$\Rightarrow U \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \text{ hvis og bare hvis } \lambda_1 = \lambda_2 = \lambda_3$$

Exempel p<sup>o</sup> bruk : Pappos i  $\mathbb{P}^2$



$A_3, B_3, C_3$  kollimare



$$A_3 = (b_1 b_2 - c_1 c_2 : b_1 - c_1 : b_2 - c_2)$$

$$B_3 = (c_1 c_2 - a_1 a_2 : c_1 - a_1 : c_2 - a_2)$$

$$C_3 = (a_1 a_2 - b_1 b_2 : a_1 - b_1 : a_2 - b_2)$$

fordi  $\begin{vmatrix} b_1 & 0 & 1 \\ x & y & z \\ c_2 & 1 & 0 \end{vmatrix} = \begin{vmatrix} c_1 & 0 & 1 \\ x & y & z \\ b_2 & 1 & 0 \end{vmatrix}$  eller  $-b_1 z + x - c_2 y = -c_1 z + x - b_2 y = 0$

$$(b_1 - c_1) z = (b_2 - c_2) y$$

$$x = b_1 z + c_2 y$$

$$= b_1(b_2 - c_2) + c_2(b_1 - c_1)$$

$$= b_1 b_2 - b_1 c_2 + c_2 b_1 - c_1 c_2$$

$$= b_1 b_2 - c_1 c_2$$

$$\begin{array}{rrr} b_1 b_2 - c_1 c_2 & b_1 - c_1 & b_2 - c_2 \\ + & c_1 c_2 - a_1 a_2 & c_1 - a_1 & c_2 - a_2 \\ + & a_1 a_2 - b_1 b_2 & a_1 - b_1 & a_2 - b_2 \\ \hline 0 & 0 & 0 \end{array}$$

(lineært avhengige)