Torsdag 15. oktober

Et lite tilbakeblikk på "polyeder-telling"



Ukjent konvekst polyeder

V hjørner hjørne i: a_i 3-kanter, b_i 4-kanter, c_i 5-kanter, d_i 6-kanter

Antall kanter: $e = \frac{1}{2}\sum_{i=1}^{V}(a_i+b_i+c_i+d_i)$

Antall flater: $f = \sum_{i=1}^{V}\left(\frac{a_i}{3}+\frac{b_i}{4}+\frac{c_i}{5}+\frac{d_i}{6}\right)$

$$\chi = v-e+f = \sum_{i=1}^{V}\left(1-\frac{1}{2}(a_i+b_i+c_i+d_i)+\left(\frac{a_i}{3}+\frac{b_i}{4}+\frac{c_i}{5}+\frac{d_i}{6}\right)\right)$$

$$= \sum_{i=1}^{V}\left(1-\frac{a_i}{6}-\frac{b_i}{4}-\frac{3c_i}{10}-\frac{d_i}{3}\right)$$

Vinkel-defekta i hjørne i: $D_i = 2\pi - a_i\frac{\pi}{3} - b_i\frac{\pi}{2} - c_i\frac{3\pi}{5} - d_i\frac{2\pi}{3}$

Samlet defekt: $D = \sum_{i=1}^{V}\left(2\pi - a_i\frac{\pi}{3} - b_i\frac{\pi}{2} - c_i\frac{3\pi}{5} - d_i\frac{2\pi}{3}\right)$

$$= 2\pi\sum_{i=1}^{V}\left(1-\frac{a_i}{6}-\frac{b_i}{4}-\frac{3c_i}{10}-\frac{d_i}{3}\right)$$

$$= 2\pi\chi$$

Som betyr $\chi = 2 \Leftrightarrow D = 4\pi$

Oppklaring fra mandag.

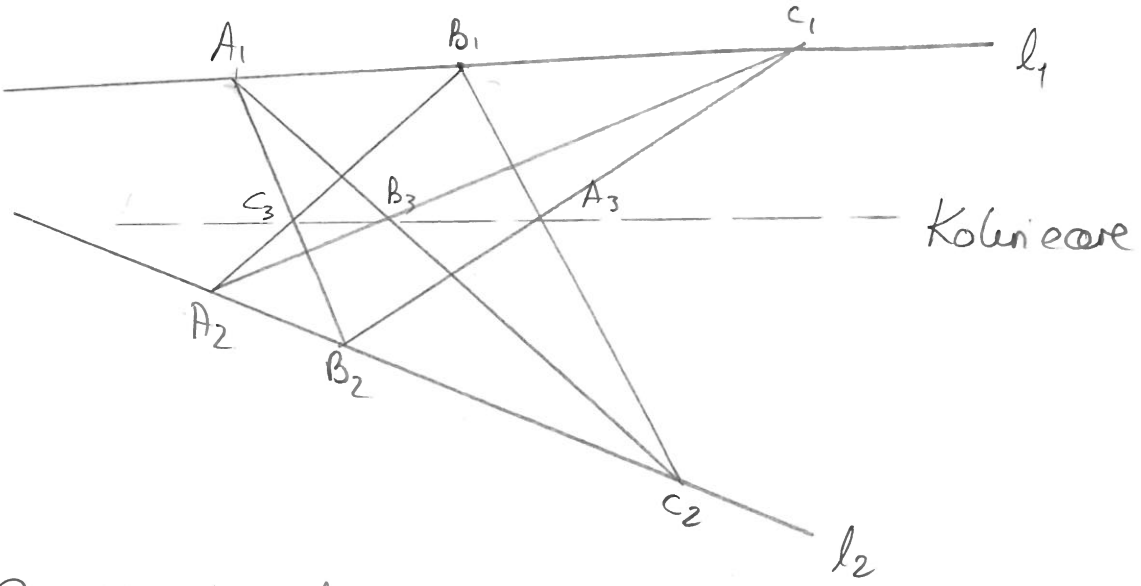
La $M = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$. $\lambda_1, \lambda_2, \lambda_3 \neq 0$

Det gir $M(1:0:0) = (\lambda_1 : 0 : 0) \sim (1:0:0)$
$M(0:1:0) = (0 : \lambda_2 : 0) \sim (0:1:0)$
$M(0:0:1) = (0 : 0 : \lambda_3) \sim (0:0:1)$

Men $M(1:1:1) = (\lambda_1 : \lambda_2 : \lambda_3)$

Som betyr at de tre punktene $(1:0:0)$, $(0:1:0)$ og $(0:0:1)$ ikke er nok for å sikre entydighet

$\Rightarrow$ Trenger 4 punkter



Projektiv transformasjon:

$A_1 = (a_1 : 0 : 1)$ $B_1 = (b_1 : 0 : 1)$ $C_1 = (c_1 : 0 : 1)$
$A_2 = (a_2 : 1 : 0)$ $B_2 = (b_2 : 1 : 0)$ $C_2 = (c_2 : 1 : 0)$

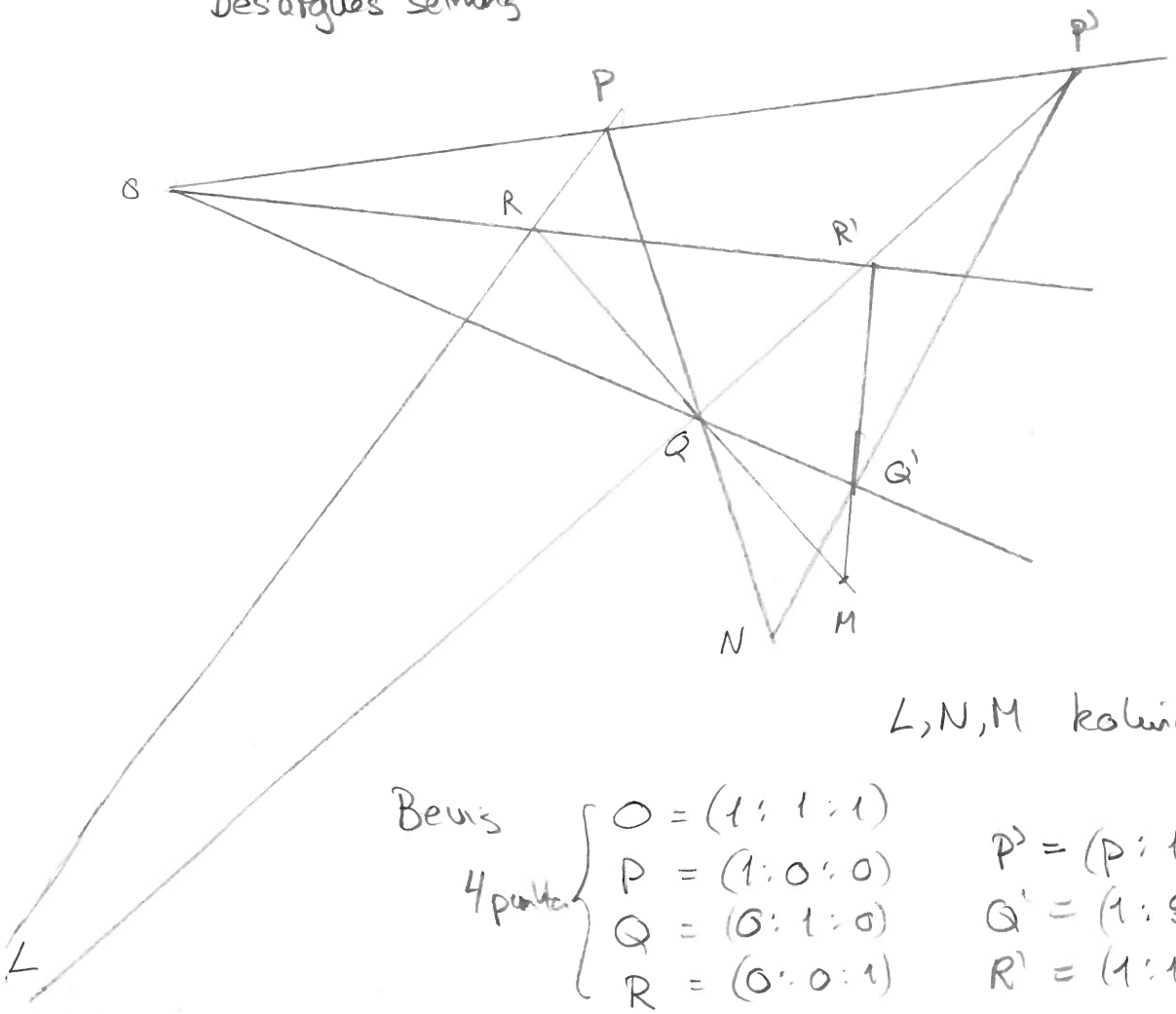
4 punkter

på l_1 og l_2

$\Rightarrow$
$A_3 = (b_1 b_2 - c_1 c_2 : b_1 - c_1 : b_2 - c_2)$
$B_3 = (c_1 c_2 - a_1 a_2 : c_1 - a_1 : c_2 - a_2)$ } på linje
$C_3 = (a_1 a_2 - b_1 b_2 : a_1 - b_1 : a_2 - b_2)$

0

Desargues setning



L, N, M kolineare

Bevis 4 punkter:

$O = (1:1:1)$ $\quad P' = (p:1:1)$

$P = (1:0:0)$ $\quad Q' = (1:q:1)$

$Q = (0:1:0)$ $\quad R' = (1:1:r)$

$R = (0:0:1)$

Punkt på OP: $\begin{vmatrix} x & y & z \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} = y - z = 0$

$PQ = \begin{vmatrix} x & y & z \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = z = 0$

$P'Q' = \begin{vmatrix} x & y & z \\ p & 1 & 1 \\ 1 & q & 1 \end{vmatrix} = (1-q)x + (1-p)y + (pq-1)z$

$N = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 1 \\ 1-q & 1-p & pq-1 \end{vmatrix} = (p-1 : 1-q : 0)$

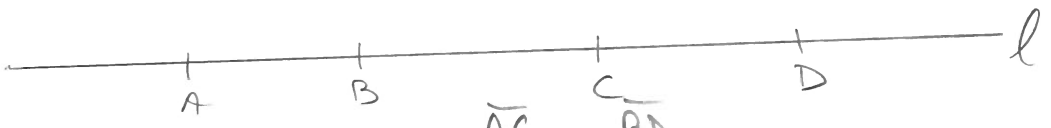
Tilsvarende $L = (p-1 : 0 : 1-r)$

$M = (0 : 1-q : r-1)$

$$\begin{vmatrix} p-1 & 1-q & 0 \\ -p+1 & 0 & r-1 \\ 0 & -1+q & -r+1 \end{vmatrix} = 0$$

Kolineære

Kryssforhold



$$(AB,CD) = \frac{\overline{AC}}{\overline{AD}} \cdot \frac{\overline{BD}}{\overline{BC}}$$

Punktene er harmonisk konjugert dersom $(AB,CD) = -1$

Eksempel



$$(AB,CD) = \frac{c}{d} \cdot \frac{d-b}{c-b}$$

$b=1$, $c=2$ Harmonisk $\frac{2}{d} \cdot \frac{d-1}{1} = \frac{2(d-1)}{d} = -1$

$\Leftrightarrow 2d-2 = -d$

ella $d = \frac{2}{3}$??

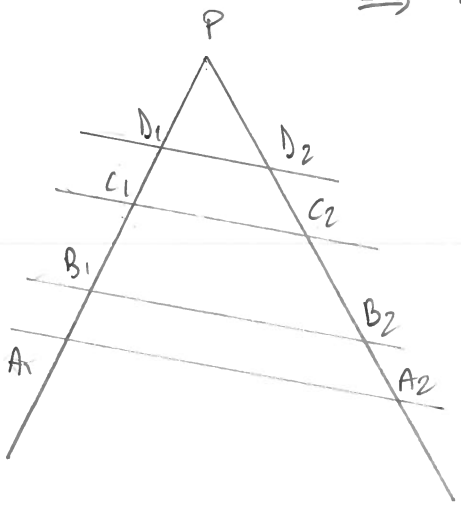
Lineæravbildning: $M = \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix}$

$A' = M\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $B' = M\begin{pmatrix} b \\ 0 \end{pmatrix} = b\begin{pmatrix} x_{11} \\ x_{21} \end{pmatrix}$ $C' = c\begin{pmatrix} x_{11} \\ x_{21} \end{pmatrix}$ $D' = d\begin{pmatrix} x_{11} \\ x_{21} \end{pmatrix}$

$A'C' = c\sqrt{x_{11}^2 + x_{21}^2}$ $A'D' = d\sqrt{x_{11}^2 + x_{21}^2}$

$B'D' = (d-b)\sqrt{x_{11}^2 + x_{21}^2}$ $B'C' = (c-b)\sqrt{x_{11}^2 + x_{21}^2}$

$\Rightarrow$ Kryssforholdet er bevart



$$\frac{PC_1}{C_1C_2} = \frac{PA_1}{A_1A_2} \qquad \frac{PC_2}{C_1C_2} = \frac{PA_2}{A_1A_2}$$

$$\frac{C_1C_2}{A_1A_2} = \frac{PC_1}{PA_1} = \frac{PC_2}{PA_2}$$

Tilsvarende for ...

$$(A_1B_1,C_1D_1) = \frac{A_1C_1}{A_1D_1} \cdot \frac{B_1D_1}{B_1C_1} = \frac{A_1P - C_1P}{A_1P - D_1P} \cdot \frac{B_1P - D_1P}{B_1P - C_1P}$$

$$= (A_2B_2, C_2D_2)$$

byttet 1/2 indeks



Harmonisk deling: $\dfrac{1}{\frac{1}{2}\left(\frac{1}{\overline{AC}}+\frac{1}{\overline{AD}}\right)} = \overline{AB}$

$$\Rightarrow \quad \frac{1}{\overline{AB}} = \frac{1}{2}\left(\frac{1}{\overline{AC}}+\frac{1}{\overline{AD}}\right)$$

$$\Leftrightarrow \quad 2 = \frac{\overline{AB}}{\overline{AC}} + \frac{\overline{AB}}{\overline{AD}} = \frac{\overline{AC}+\overline{CB}}{\overline{AC}} + \frac{\overline{AD}+\overline{DB}}{\overline{AD}}$$

$$= 1 + \frac{\overline{CB}}{\overline{AC}} + 1 + \frac{\overline{DB}}{\overline{AD}}$$

$$\Leftrightarrow \quad \frac{\overline{CB}}{\overline{AC}} + \frac{\overline{DB}}{\overline{AD}} = 0 \quad \text{eller} \quad \overline{CB}\cdot\overline{AD} = -\overline{DB}\cdot\overline{AC} = \overline{BD}\cdot\overline{AC}$$

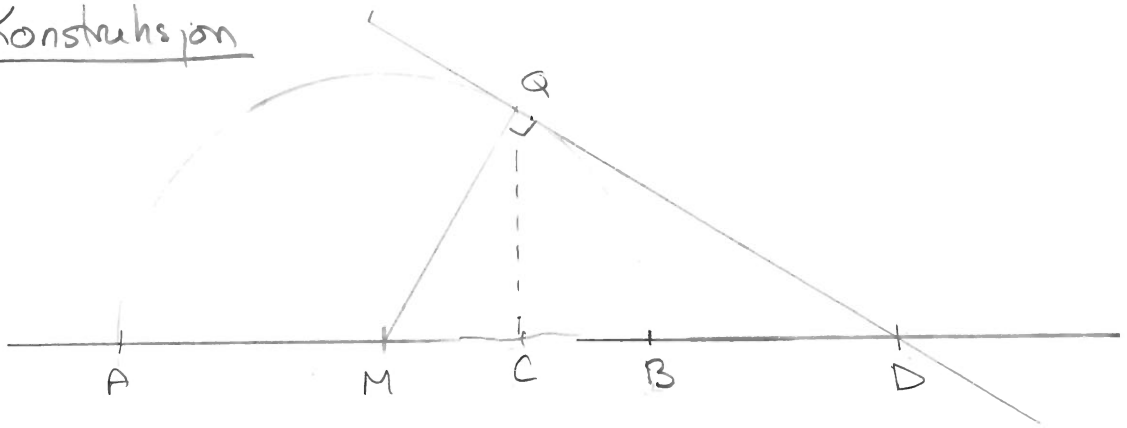
Som betyr $(AB,CD) = \dfrac{\overline{AC}}{\overline{AD}}\cdot\dfrac{\overline{BD}}{\overline{BC}} = \dfrac{\overline{CB}\cdot\overline{AD}}{\overline{AD}\cdot\overline{BC}} = -1$

Forholdet mellom harmonisk deling og gjennomsitt:

$$\frac{1}{x} = \frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}\right) \qquad y = \frac{1}{2}(a+b)$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{2}\,\frac{a+b}{ab} = \frac{y}{ab} \quad \text{dvs} \quad \underline{xy=ab}$$

Konstruksjon



M : midtpunktet på AB

Q : tangeringspunktet på sirkelen med radius AM og sentrum i M for linja gjennom D.

C : fotpunktet fra Q på AB

$\triangle MCQ \sim \triangle MQD \Rightarrow \frac{MC}{MQ} = \frac{MQ}{MD}$ eller $MC \cdot MD = MQ^2$

omskrives til $MC \cdot MD = AM \cdot MB = -MA \cdot MB$

Skal vise $(AB,CD) = \frac{AC}{AD} \cdot \frac{BD}{BC} = -1$ dvs $AC \cdot BD = -AD \cdot BC$

$$AC \cdot BD + AD \cdot BC = (AM+MC)(BM+MD) + (AM+MD)(BM+MC)$$

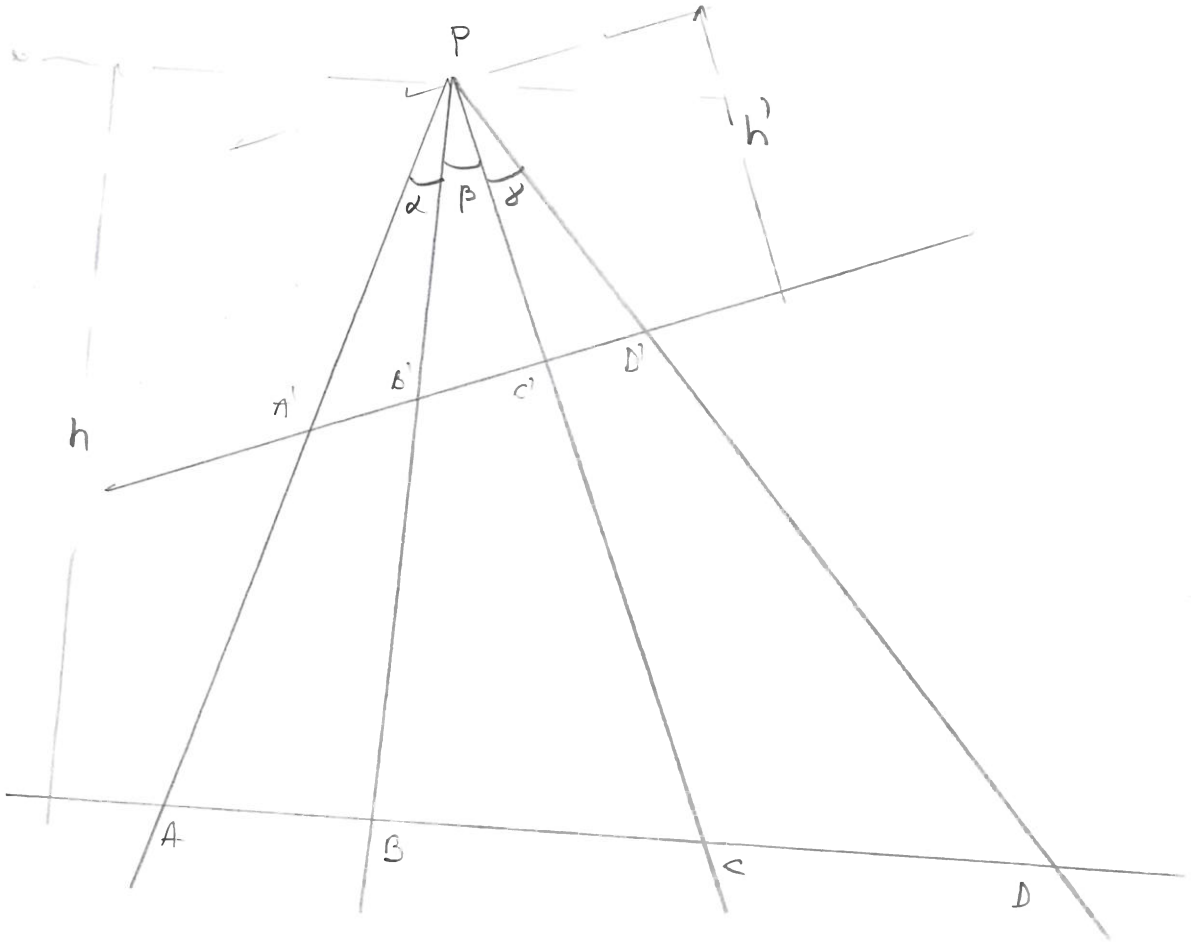
$$= AM \cdot BM + AM \cdot MD + MC \cdot BM + MC \cdot MD \quad (MC \cdot MD = -MA \cdot MB)$$

$$+ AM \cdot BM + AM \cdot MC + MD \cdot BM + MD \cdot MC$$

$$= AM \cdot MD + MC \cdot BM + AM \cdot MC + MD \cdot BM$$

$$= (AM + BM)(MD + MC) = 0$$

$AM + BM = 0$



$$\frac{1}{2} AC \cdot h = \frac{1}{2} AP \cdot CP \cdot \sin(\alpha+\beta)$$

$$\frac{1}{2} AD \cdot h = \frac{1}{2} AP \cdot DP \cdot \sin(\alpha+\beta+\gamma)$$

$$\frac{1}{2} BC \cdot h = \frac{1}{2} BP \cdot CP \cdot \sin\beta$$

$$\frac{1}{2} BD \cdot h = \frac{1}{2} BP \cdot DP \cdot \sin(\beta+\gamma)$$

$$\Rightarrow \frac{AC}{AD} \cdot \frac{BD}{BC} = \frac{AP \cdot CP \cdot \sin(\alpha+\beta)}{AP \cdot DP \cdot \sin(\alpha+\beta+\gamma)} \cdot \frac{BP \cdot DP \cdot \sin(\beta+\gamma)}{BP \cdot CP \cdot \sin\beta}$$

$$= \frac{\sin(\alpha+\beta)\sin(\beta+\gamma)}{\sin(\alpha+\beta+\gamma) \cdot \sin\beta}$$