Torsdag 29. oktober

Eksamen H18 oppg. 3

$\mathbb{P}^2 : (x_0 : x_1 : x_2)$

$\mathbb{R}^2 \subset \mathbb{P}^2$
$x_2 \neq 0$

$Q_a \subset \mathbb{P}^2 ; \; x_0^2 + a x_1^2 - x_2^2 = 0$

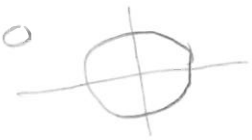
a) $a = -1$ $\quad x_0^2 - x_1^2 - 1 = 0$



$a = 0$ $\quad x_0^2 - 1 = 0$



$a = 1$ $\quad x_0^2 + x_1^2 - 1 = 0$



b) $Q_a \cap \{x_2 = 1\}$ gitt ved $x_0^2 + a x_1^2 - 1 = 0$

Ellipse : $a > 0$
Hyperbel : $a < 0$

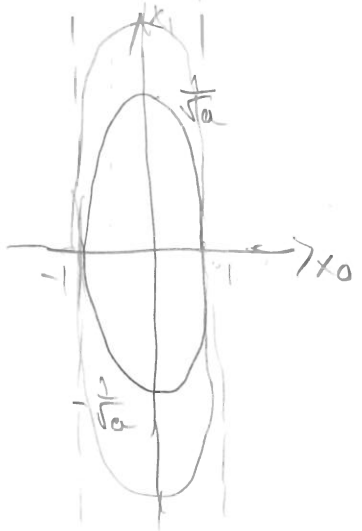
c) Q_a degenerert : $a = 0$ (eller a))

$x_0^2 - x_2^2 = 0$ dvs $x_0 - x_2 = 0$ eller $x_0 + x_2 = 0$

Svarer til to linjer

$a > 0$ $\quad x_0^2 + a x_1^2 - x_2^2 = 0$ Ingen punkter i ∞

$x_2 = 1$: $\quad x_0^2 + a x_1^2 = 1$, eller $x_0^2 + \dfrac{x_1^2}{\left(\frac{1}{\sqrt{a}}\right)^2} = 1$



- Samme bredde
- Krøyder ofher

$\Rightarrow$ Ellipsen strekkes ut og nærmer seg to rette linjer

d) $P = (1:0:1) \in \mathbb{P}^2$
$Q = (1:1:1)$

$L = PQ :$ $\begin{vmatrix} x_0 & x_1 & x_2 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -x_2 + x_0 = 0$

$L \cap Q_a$: $\{x_0 - x_2 = 0\} \cap \{x_0^2 + a x_1^2 - x_2^2 = 0\}$

$\Rightarrow \quad x_0^2 + a x_1^2 - x_0^2 = a x_1^2 = 0$

Dobbelt punkt for $a \neq 0$

dvs linja er tangent til Q_a

e) $R = L \cap \{x_2 = 0\} = (0:1:0)$

$L' \ni R$, L' tangerer Q_a

$\forall a \in \mathbb{R}$

Linje gjennom $(0;1:0)$: $p x_0 + q x_1 + r x_2 = 0$

$\Rightarrow q = 0$

dvs $p x_0 + r x_2 = 0$ L'

Snitt $L' \cap Q_a$: $p x_0 + r x_2 = 0$ $x_0^2 + a x_1^2 - x_2^2 = 0$

<u>$p = 0, r \neq 0$</u> $\Rightarrow x_2 = 0$ og $x_0^2 + a x_1^2 = 0$. Ingen løsning

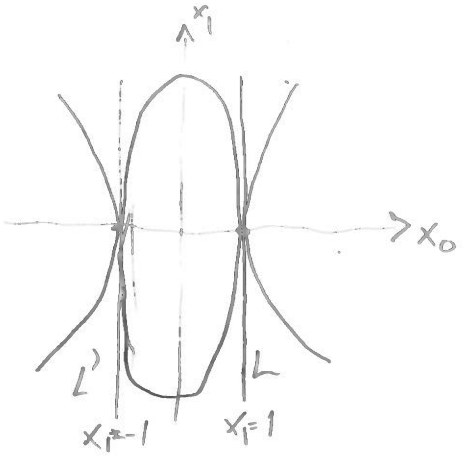
<u>$p \neq 0$</u> $x_0 = -\frac{r}{p} x_2$ og $\left(\frac{r^2}{p^2} - 1\right) x_2^2 + a x_1^2 = 0$

$\Delta = -4a\left(\frac{r^2}{p^2} - 1\right)$

Tangent for $a = 0$ eller $\frac{r^2}{p^2} = 1$

$r^2 = p^2$ L' gitt ved $x_0 \pm x_2 = 0$

dvs $x_0 - x_2 = 0$ gir L, $x_0 + x_2 = 0$ gir L'



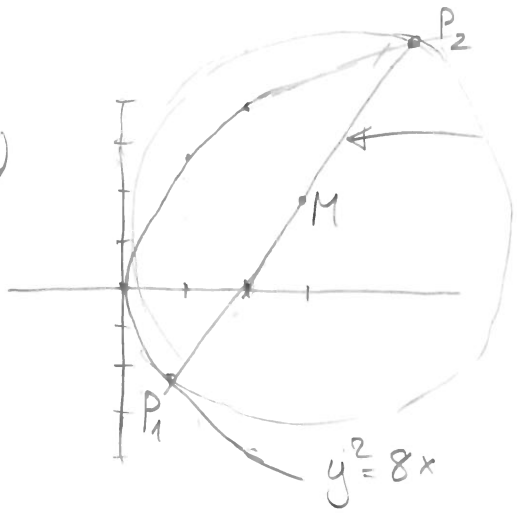
Eksamen H15 oppgave 3

$y^2 = 8x$

Korde gjennom (2,0), stigningstall k

Korden er diameter i en sirkel

a)



Ett punkts formel

$y = k(x-2)$

$k=1$:

$y = x-2$

P_1, P_2: $(x-2)^2 = 8x$ eller $x^2 - 12x + 4 = 0$

$$\Rightarrow x = \frac{12 \pm \sqrt{144-16}}{2} = \frac{12 \pm \sqrt{128}}{2}$$

$$= 6 \pm 4\sqrt{2}$$

$$y = 4 \pm 4\sqrt{2}$$

$$M = \frac{P_1+P_2}{2} = \frac{1}{2}\left((6+4\sqrt{2}, 4+4\sqrt{2}) + (6-4\sqrt{2}, 4-4\sqrt{2})\right)$$

$$= (6,4) \quad \text{Radius } \sqrt{(4\sqrt{2})^2 + (4\sqrt{2})^2} = 8$$

$$\Rightarrow (x-6)^2 + (y-4)^2 = 64$$

P_1P_2: $y - (4+4\sqrt{2}) = \frac{4+4\sqrt{2}-(4-4\sqrt{2})}{6+4\sqrt{2}-(6-4\sqrt{2})}\left(x - (6+4\sqrt{2})\right)$

$$y = x - (6+4\sqrt{2}) + (4+4\sqrt{2}) = x - 6 - 4\sqrt{2} + 4 + 4\sqrt{2} = x-2$$

Gjennom (2,0) stigningstall $k=1$.

b) Vilkårlig k: P_1, P_2: $(k(x-2))^2 = 8x$ eller $k^2x^2 - (4k^2+8)x + 4k^2 = 0$

$$\Rightarrow x = \frac{4k^2+8 \pm \sqrt{16k^4+64k^2+64-16k^4}}{2k^2}$$

$$= 2 + \frac{4}{k^2} \pm 4\frac{\sqrt{k^2+1}}{k^2}$$

$$y = k(x-2) = \frac{4}{k} \pm 4\frac{\sqrt{k^2+1}}{k}$$

$$M = \frac{P_1+P_2}{2} = \left(2 + \frac{4}{k^2}, \frac{4}{k}\right) \quad \text{Radius } \sqrt{\left(4\frac{\sqrt{k^2+1}}{k^2}\right)^2 + \left(4\frac{\sqrt{k^2+1}}{k}\right)^2} =$$

$$= \sqrt{\frac{16}{k^4}(k^2+1) + \frac{16}{k^2}(k^2+1)}$$

$$= \sqrt{\frac{16}{k^4}(k^4+2k^2+1)} = \frac{4(k^2+1)}{k^2}$$

Sirkel $\left(x-\left(2+\frac{4}{k^2}\right)\right)^2+\left(y-\frac{4}{k}\right)^2=\left(\frac{4(k^2+1)}{k^2}\right)^2$

$k=1$ gir $(x-6)^2+(y-4)^2=64$.

c) Linja $x=-2$ skjærer sirkelen

$$\left(-2-\left(2+\frac{4}{k^2}\right)\right)^2+\left(y-\frac{4}{k}\right)^2=\left(\frac{4(k^2+1)}{k^2}\right)^2$$

ella $\left(\frac{4(k^2+1)}{k^2}\right)^2+\left(y-\frac{4}{k}\right)^2=\left(\frac{4(k^2+1)}{k^2}\right)^2$

dvs $\left(y-\frac{4}{k}\right)^2=0$ Dobbelt punkt, altså tangent

d) Midtpunktet: $M=\left(2+\frac{4}{k^2},\frac{4}{k}\right)=(x,y)$

$\Rightarrow \quad x-2=\frac{4}{k^2} \quad y^2=\frac{16}{k^2}=4(x-2)$

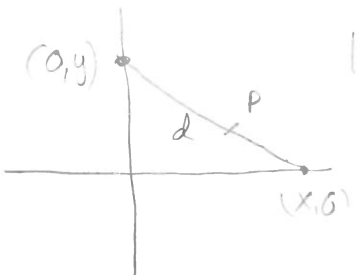
Altså $\underline{y^2=4(x-2)}$

e) $k\to\infty \quad x-2\to 0 \quad y\to 0$ dvs $(2,0)$

$k\to 0 \quad x-2\to\infty \quad y\to\infty$

$$\frac{y}{x}=\frac{\frac{4}{k}}{\frac{4}{k^2}+2}=\frac{4k}{4+2k^2}\to 0$$

Oppg 26 s.140

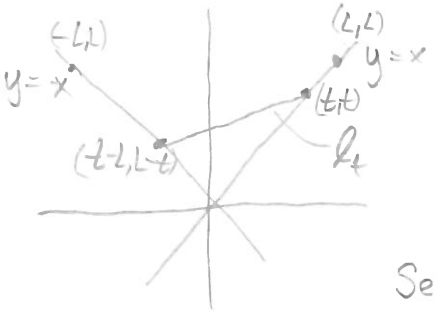


$\|(x,0)-(0,y)\|=\sqrt{x^2+y^2}=d$

$P=t(x,0)+(1-t)(0,y)=(tx,(1-t)y)=(u,v)$

$\left(\frac{u}{t}\right)^2+\left(\frac{v}{1-t}\right)^2=d^2$ ellipse

Oppg 29 s. 140



$\exists$ parabel slik at alle l_t tangerer

Symmetri gir at parabelen må ha formen $y = \alpha x^2 + \beta$

Sett $t = \frac{L}{2}$ $l_{\frac{L}{2}}$: $y = \frac{L}{2}$ $x = 0$ tangerer

$\Rightarrow \beta = \frac{L}{2}$

Generelt:

$$l_t: \quad y - t = \frac{t-(L-t)}{t-(t-L)}(x-t) = \frac{2t-L}{L}(x-t)$$

skjæring med parabel:

$$\alpha x^2 + \frac{L}{2} - t = \frac{2t-L}{L}x - \frac{2t-L}{L}t$$

eller

$$\alpha x^2 - \frac{2t-L}{L}x + \underbrace{\frac{L}{2} - t + \frac{2t-L}{L}t}_{\frac{1}{2L}(L^2-2Lt+4t^2-2Lt)} = 0$$

$$= \frac{1}{2L}(L^2-4Lt+4t^2) = \frac{1}{2L}(L-2t)^2$$

l_t er tangent hvis $D = 0$

$$D = \left(\frac{2t-L}{L}\right)^2 - 4\alpha\frac{1}{2L}(L-2t)^2$$

$$= \frac{(2t-L)^2}{L^2} - \frac{2\alpha L}{L^2}(L-2t)^2 = \frac{(L-2t)^2}{L^2}(1-2\alpha L) = 0$$

dvs $\alpha = \frac{1}{2L}$; $y = \frac{x^2}{2L} + \frac{L}{2}$ tangerer alle l_t.