UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: MAT3100 — Linear optimization.

Day of examination: Wednesday 11. June 2025.

Examination hours: 09:00 – 13:00.

This problem set consists of 2 pages.

Appendices: None.

Permitted aids: None.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

All 10 part questions will be weighted equally.

Problem 1 (Simplex method). We will consider the following linear programming problem.

$$\begin{array}{lrcrcrcl} \max & 4x_1 & + & 2x_2 & + & x_3 & & \\ \text{s.t.} & x_1 & + & x_2 & + & x_3 & \leq & 1 \\ & \frac{1}{2}x_1 & - & x_2 & & & \leq & 0 \\ & -2x_1 & + & x_2 & & & \leq & 0 \\ & & & & & x_1, x_2, x_3 & \geq & 0 \end{array} \tag{1}$$

a) Write down the dictionary corresponding to (1), and solve the problem using the simplex method and the largest coefficient rule. Write also down the optimal value and optimal solution. Is the optimal solution unique?

b) What does it mean for a dictionary to be degenerate, and for a pivot to be degenerate? Which dictionaries/pivots in **a)** were degenerate?

c) Write down the dual problem and the optimal dual dictionary. What is the optimal solution to the dual problem? Is this solution unique?

d) What is the optimal solution if the objective function in (1) is changed to $6x_1 + 3x_2 + 5x_3$? Is the optimal solution unique now?

e) How is an extreme point of a convex set defined? Find at least three extreme points of the feasible region of the problem (1).

Problem 2 (Convexity). Let $C \subseteq \mathbb{R}^{n+1}$ be a convex set, and let $D \subseteq \mathbb{R}^n$ be the set

$$D = \{\mathbf{x} \in \mathbb{R}^n : (\mathbf{x}, a) \in C \text{ for some } a \in \mathbb{R}\}.$$

D is also called the *projection* of C onto $\mathbb{R}^n$. Show that D is also convex.

Problem 3 (Game theory). Consider the matrix game with payoff

(Continued on page 2.)

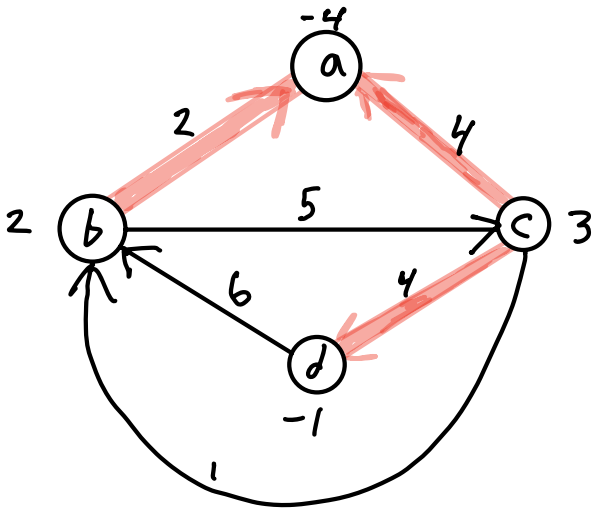


Figure 1: Flow problem for Problem 4.

matrix

$$A = \begin{pmatrix} -2 & -3 & -1 \\ -3 & -1 & 0 \\ -1 & -1 & -2 \end{pmatrix}.$$

a) Find the pure minmax and maxmin strategies of the game. Does the matrix A have a saddle point? What does this say about the possibility of having pure strategies which are optimal?

b) Show that the randomised strategies $x^* = (1/2, 0, 1/2)$ and $y^* = (1/2, 0, 1/2)$ are mutually optimal for the column player and row player. What is the value of the game? Is the game fair?

Problem 4 (Network flow). Consider the minimum cost network flow problem based on the directed graph shown in Figure 1. The number associated with each directed edge is the cost per unit flow, and the number associated with each node is the supply at that node.

a) Let T_1 be the spanning tree consisting of the edges (b, a), (c, a), and (c, d) (indicated in bold in the figure). Compute the tree solution corresponding to T_1.

b) Use the network simplex method to find an optimal solution and optimal value for the flow problem.

Good luck!