UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Examination in	INF-MAT 3370 — Linear optimization
Day of examination:	3. juni 2008
Examination hours:	14.30 – 17.30

This problem set consists of 3 pages.

Appendices:	None
Permitted aids:	Only approved calculator.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

There are 9 questions with about the same weight.

Problem 1

Consider the LP problem:

$$\begin{array}{lrcrcl} \max & -x_1 & + & 2x_2 & & \\ \text{subject to} & & & & & \\ & -x_1 & + & x_2 & \leq & 3 \\ & x_1 & & & \leq & 5 \\ & & & x_2 & \leq & 5 \\ & & & x_1, x_2 \geq 0. & & \end{array} \tag{1}$$

1a

Solve problem (1) by the simplex algorithm with initial solution $(x_1, x_2) = (0, 0)$. Find an optimal solution and the corresponding optimal value.

1b

Illustrate problem (1) geometrically in the plane: draw the feasible region and the level curve of the objective function $f(x_1, x_2) = -x_1 + 2x_2$ in the optimal solution.

1c

Is there a unique optimal solution of (1)? Explain your answer. Also find an optimal dual solution (i.e., an optimal solution of the dual problem of (1)).

(Continued on page 2.)

Problem 2

Consider the following LP problem given by the dictionary:

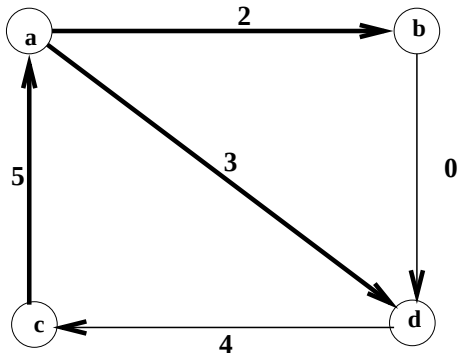
$$\begin{array}{rrcrcrcr} \zeta = & 10 & - & x_1 & - & x_2 & - & 2x_3 \\ \hline x_4 = & -1 & & & + & x_2 & - & 4x_3 \\ x_5 = & 2 & - & x_1 & + & x_2 & & \\ x_6 = & 0 & + & 3x_1 & & & - & x_3 \end{array} \tag{2}$$

2a

Solve problem (2) by the dual simplex algorithm and give an optimal solution and the optimal value.

Problem 3

We have a network flow problem in the following directed graph with vertices (nodes) a, b, c, d:



The numbers along each edge indicates its cost c, and the bold lines (edges) (a, b), (a, d) og (c, a) represent a spanning tree which corresponds to a basis in the problem.

3a

Compute the corresponding *dual* basic solution (y and z) where we let $y_d = 0$. Is this an optimal basis? Explain your answer.

3b

What can you say in general about the determinant of an LP basis matrix in a network flow problem?

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Problem 4

Let $c_1, c_2, \ldots, c_n$ be given real numbers, and define the function $f : \mathbb{R}^n \to \mathbb{R}$ by

$$f(x) = \sum_{j=1}^{n} c_j x_j + \sum_{j=1}^{n} |x_j|.$$

where $x = (x_1, x_2, \ldots, x_n)$. Let A be an $m \times n$ matrix and $b \in \mathbb{R}^m$. Consider the minimization problem

$$\begin{array}{ll} \min & f(x) \\ \text{subject to} & \\ & Ax \leq b. \end{array} \tag{3}$$

4a

Write (3) as an LP problem, and argue why this construction is correct.

4b

Show that the function f is convex.

4c

Let now $P \subseteq \mathbb{R}^n$ be a polytope. Show that there is an optimal solution x^* of the problem

$$\max\{f(x) : x \in P\}$$

where x^* is an extreme point of P.

Good luck!