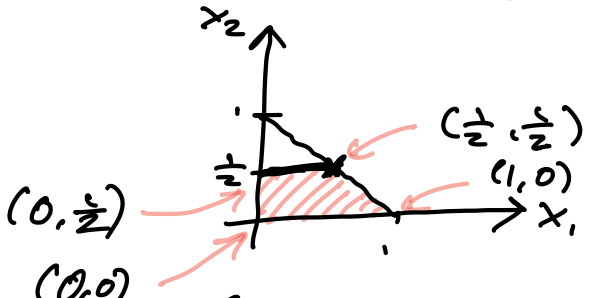
Exercise 20

$$P = \{ x : x_1 + x_2 \le 1 \,,\; x_2 \le \tfrac{1}{2} \,,\; x_1, x_2 \ge 0 \}$$



$$\begin{aligned} x_1 + x_2 + w_1 &= 1 \\ x_2 + w_2 &= \tfrac{1}{2} \end{aligned} \quad \Rightarrow \quad A = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

all but $\{1,3\}$ are possible bases

$B = \{1,2\}$: x_1, x_2 basic $\Rightarrow$ w_1, w_2 nonbasic $\Rightarrow$ $w_1 = w_2 = 0$

$\Downarrow$ $\qquad$ $\Downarrow$

$x_1 + x_2 = 1$ $\qquad$ $x_2 = \frac{1}{2} \Rightarrow x_1 = \frac{1}{2}$

$\Downarrow$

$(\frac{1}{2}, \frac{1}{2})$, feasible.

$B = \{1,4\}$: x_1, w_2 are basic $\Rightarrow$ x_2, w_1 nonbasic $\Rightarrow$ $x_2 = w_1 = 0$

$\Downarrow$

$x_1 + x_2 = 1$

$\Downarrow$

$x_1 = 1$

$\Downarrow$

$(1,0)$, feasible. $\qquad$ $(w_2 = \frac{1}{2} - x_2 = \frac{1}{2})$

$B = \{2,3\}$, x_2, w_1 basic $\Rightarrow$ x_1, w_2 nonbasic $\Rightarrow$ $x_1 = w_2 = 0$

$\Downarrow$ $\qquad$ $w_1 = 1 - x_1 - x_2 = \frac{1}{2}$

$x_2 = \frac{1}{2}$

$(0, \frac{1}{2})$ feasible

$B = \{2,4\}$: x_2, w_2 basic $\Rightarrow$ x_1, w_1 nonbasic $\Rightarrow x_1 = w_1 = 0$

$\Downarrow$

$x_1 + x_2 = 1 \Rightarrow x_2 = 1$ $\quad w_2 = \frac{1}{2} - x_2 = -\frac{1}{2} < 0$

$\Downarrow$

$(0,1)$ infeasible.

$B = \{3,4\}$: w_1, w_2 basic $\Rightarrow$ x_1, x_2 nonbasic $\Rightarrow x_1 = x_2 = 0$ $\quad w_1 = 1$, $w_2 = \frac{1}{2}$

$\Downarrow$

$(0,0)$ feasible.

Exercise 21 $\quad P = \{x : x_1 + x_2 + x_3 \le 4,\ x_1 + x_2 \ge 1,\ x_3 \le 2,\ x_i \ge 0\}$

$$\left.\begin{aligned} x_1 + x_2 + x_3 + w_1 &= 4 \\ -x_1 - x_2 + w_2 &= -1 \\ x_3 + w_3 &= 2 \end{aligned}\right\} \quad A = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Possible bases: $\{1,3,4\}, \{1,3,5\}, \{1,3,6\}, \{1,4,6\}, \{1,5,6\}, \ldots$

$B = \{1,3,4\}$: x_1, x_3, w_1 basic $\Rightarrow$ x_2, w_2, w_3 nonbasic $\Rightarrow x_2 = w_2 = w_3 = 0$

$\Downarrow$

$x_1 + x_2 = 1 \Rightarrow x_1 = 1$

$x_3 = 2$

$\downarrow$

$(1,0,2)$ feasible

$w_1 = 4 - x_1 - x_2 - x_3 = 4 - 1 - 2 = 1$

$B = \{1,3,6\}$: x_1, x_3, w_3 basic $\Rightarrow$ x_2, w_1, w_2 nonbasic $\Rightarrow x_2 = w_1 = w_2 = 0$

$x_1 + x_2 + x_3 = 4$ $\qquad x_1 + x_2 = 1$

$\downarrow$

$x_3 = 4 - 1 = 3$

$w_3 = 2 - x_3 = 2 - 3 = -1$

$(1, 0, 3)$, infeasible.

Exercise 22

Fourier-Motzkin on

$$x_1, x_2 \geq 0$$
$$x_1 + x_2 \leq 1$$
$$x_2 \leq \frac{1}{2}$$

$I_+ : x_1 \leq 1 - x_2$

$I_- : 0 \leq x_1$

$I_0 : x_2 \geq 0, \; x_2 \leq \frac{1}{2}$

$$0 \leq x_1 \leq 1 - x_2$$
$$0 \leq x_2 \leq \frac{1}{2}$$

Exercise 23

Fourier-Motzkin on

$$x_1, x_2, x_3 \geq 0$$
$$x_1 + x_2 + x_3 \leq 4$$
$$x_1 + x_2 \geq 1$$
$$x_3 \leq 2$$

$I_+ : x_1 \leq 4 - x_2 - x_3$

$I_- : 0 \leq x_1, \; 1 - x_2 \leq x_1$

$I_0 : x_2, x_3 \geq 0, \; x_3 \leq 2$

$$\max(0, 1 - x_2) \leq x_1 \leq 4 - x_2 - x_3$$
$$x_2, x_3 \geq 0$$
$$x_3 \leq 2$$

$I_- : 0 \leq x_2$

$I_0 : 0 \leq x_3 \leq 2$

Final form:

$$\max(0, 1 - x_2) \leq x_1 \leq 4 - x_2 - x_3$$
$$0 \leq x_2$$
$$0 \leq x_3 \leq 2$$

Exercise 24

$$a_{11}x_1 + a_{12}x_2 \leq b_1$$
$$a_{21}x_1 + a_{22}x_2 \leq b_2$$
$$a_{31}x_1 + a_{32}x_2 \leq b_3$$

$i \in I_+ : x_1 \leq \frac{1}{a_{i1}}(b_i - a_{i2}x_2)$

$j \in I_- : x_1 \geq \frac{1}{a_{j1}}(b_j - a_{j2}x_2)$

$a_{u1}x_1 + a_{u2}x_2 \leq b_u$

$$\max_{j \in I_-} \frac{1}{a_{j1}}(b_j - a_{j2}x_2) \leq x_1 \leq \min_{i \in I_+} \frac{1}{a_{i1}}(b_i - a_{i2}x_2)$$

