Oblig 1 MAT3100

Deadline 13. march 2025, 1430

February 24, 2025

Consider the problem

$$\begin{array}{rcrcl} \max x_1 & + & 3x_2 & & \\ \text{s.t. } -x_1 & + & x_2 & \leq & 2 \\ x_1 & - & x_2 & \leq & 2 \\ x_1 & & & \leq & 4 \\ & & x_2 & \leq & 4 \\ & & x_1, x_2 & \geq & 0 \end{array} \tag{1}$$

a)

Write down a 4×2-matrix A and vectors $\mathbf{b}$ and $\mathbf{c}$ so that (1) can be written on the form

$$\begin{aligned} \max \ & \mathbf{c}^T \mathbf{x} \\ \text{s.t. } & A\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

b)

Solve the problem using the Simplex method and the largest coefficient rule.

c)

Sketch the feasible region of the problem (1). In your drawing, indicate the (directed) path taken by the basic feasible solutions in the iterations of simplex from **b)**. If you instead applied Blands rule, what would this path be?

d)

We consider the same constraints as in **a)**, but change the objective function to x_2. Find an optimal solution to this modified problem. Is the optimal solution unique? If not, find an expression for the general optimal solution. You can use your drawing of the feasible region from **c)** as a guide.

e)

Write down the dual problem of (1), as well as the initial dual dictionary. Is this dictionary feasible?

f)

Write down the dual dictionaries corresponding to the primal dictionaries you obtained when applying the simplex method in **b)**. What are the basic solutions obtained by these dual dictionaries?

g)

Find the slack variables for the primal and dual optimal solutions. State what is meant by complementary slackness, and verify that this property holds for the optimal solutions you have found.