# Oblig 2 MAT3100

Deadline 24. march 2025, 1430

April 7, 2025

This compulsory exercise consists of two problems, with 3 subproblems each. Note that you will pass as long as you hand in the exercise on canvas. Feedback will be given on all solutions you submit.

## Problem 1 (Game theory)

For this first problem you may need to consult the lecture notes on game theory from 25. march (see the schedule of the course), as the material on pure minmax- and maxmin strategies can not be found in the book. We consider the matrix game with payoff matrix

$$A = \begin{pmatrix} 0 & 1 & -2 \\ 2 & -2 & -2 \\ 1 & -2 & 0 \end{pmatrix}$$

### a)

Find the best pure strategies for the row- and column player (i.e., the minmax and maxmin strategies), and their corresponding payoffs ($V^*$ and $V_*$). Show that the game does not have a value.

### b)

State the Minimax Theorem for matrix games. Explain why the pure strategies found above are not optimal, so that both players should consider using a randomised strategy instead.

### c)

Let $\mathbf{x}^* = (3/4, 1/4, 0)$ and $\mathbf{y}^* = (3/4, 0, 1/4)$ be randomised strategies for the column player and row player, respectively. Prove that these are optimal strategies, and find the value of the game. Is the game fair? If not, which player wins in the long run?

## Problem 2 (Convexity)

### a)

Explain what a polyhedron is, and what a polytope is. Show also that the intersection of two polyhedra is again a polyhedron, and that the intersection

of two polytopes is again a polytope.
In the following, $A + B$ is the set consisting of all points on the form $a + b$ with $a$ in the set $A$, $b$ in the set $B$.

## b)

Show that $A + B$ is a convex set when $A$ and $B$ are convex sets.

## c) (difficult)

Explain what the convex hull of a set is. Also, prove that

$$\text{conv}(A + B) = \text{conv}(A) + \text{conv}(B)$$

when $A$ and $B$ are subsets of $\mathbb{R}^n$ (conv denotes convex hull).
Hint: Consider the sum $\sum_{j,k} \lambda_j \mu_k (a_j + b_k)$ where $a_j \in A$, $b_k \in B$ and $\lambda_j \geq 0$, $\mu_k \geq 0$ and $\sum_j \lambda_j = 1$ and $\sum_k \mu_k = 1$.