Oblig 2 – MAT3100

Deadline 16. April 2025, 14.30

Exercise 1.

Consider the LP problem

$$\begin{array}{lrcrcl} \max & 2x_1 & + & 3x_2 & & \\ \text{s.t.} & & & & & \\ & -x_1 & + & x_2 & \geq & -2 \\ & x_1 & - & x_2 & \leq & 4 \\ & & & x_2 & \geq & 0 \end{array} \tag{1}$$

and note that x_1 is a "free" variable (arbitrary real number).

a)

Explain why problem (1) is equivalent to

$$\begin{array}{lrcrcl} \max & 2x_1 & + & 3x_2 & & \\ \text{s.t.} & & & & & \\ & x_1 & - & x_2 & \leq & 2 \\ & & & x_2 & \geq & 0 \end{array} \tag{2}$$

Hint: Multiply one inequality by ...., and note that one inequality is redundant.

b)

Explain why problem (2) is equivalent to

$$\begin{array}{lrcrcrcl} \max & 2x_1' & & -2x_1'' & + & 3x_2 & & \\ \text{s.t.} & & & & & & & \\ & x_1' & & - & x_1'' & - & x_2 & \leq & 2 \\ & & x_1', x_1'', x_2 & \geq & & 0 & & \end{array} \tag{3}$$

Show that problem (2) is unbounded. Do this directly without the simplex algorithm. Then solve (3) with the simplex algorithm (this problem is in our standard form).

c)

Consider the LP:

$$\max\ c^T x, \text{ subject to } \ Ax \leq b,$$

where A is an $m \times n$ matrix, and $c \in \mathbb{R}^n$, $b \in \mathbb{R}^m$. Show that this problem is equivalent to

$$\max\ c^T x^1 - c^T x^2, \text{ subject to } \ Ax^1 - Ax^2 \leq b,\ x^1, x^2 \geq 0,$$

which again can be written as

$$\max \begin{bmatrix} c \\ -c \end{bmatrix}^T \begin{bmatrix} x^1 \\ x^2 \end{bmatrix}, \text{ subject to } \begin{bmatrix} A & -A \end{bmatrix} \begin{bmatrix} x^1 \\ x^2 \end{bmatrix} \leq b,\ x^1, x^2 \geq 0,$$

Exercise 2.

Consider the LP given by the dictionary

$$\begin{array}{rrcrcrcr} \zeta = & 10 & - & 7x_1 & & & + & 2x_3 \\ \hline x_4 = & 1 & & & + & 3x_2 & - & 4x_3 \\ x_5 = & 2 & - & x_1 & + & x_2 & & \\ x_6 = & 0 & + & 3x_1 & & & - & x_3 \end{array} \tag{4}$$

a)

Solve the LP using the simplex algorithm; find an optimal solution and corresponding optimal value.

b)

Find all optimal solutions of (4). Give reasons for your answer.

Exercise 3.

Consider the LP

$$\begin{array}{lrcrcrcl} \max & -7x_1 & & & + & 2x_3 & & \\ \text{subject to} & & & & & & & \\ & & - & x_2 & + & x_3 & \leq & 1 \\ & x_1 & + & x_2 & + & x_3 & \leq & 2 \\ & & & x_1, x_2, x_3 \geq 0. & & & & \end{array}$$

a)

Write down the dual problem. Show that $x = (0, 1/2, 3/2)$ is optimal in the primal problem and $y = (1, 1)$ is optimal in the dual problem.