UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Exam in:	MAT3110/MAT4110 — Introduction to numerical analysis
Day of examination:	17 December 2025
Examination hours:	0900 – 1300
This problem set consists of 3 pages.	
Appendices:	None
Permitted aids:	None

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

All 10 part questions will be weighted equally.

Problem 1 Lower triangular matrix

Let A be a non-singular lower triangular $n \times n$ matrix.

1a

Write a Matlab (or python) routine `function x=forwardsolve(A,b)` that returns the solution $\mathbf{x} \in \mathbb{R}^n$ to the linear system $A\mathbf{x} = \mathbf{b}$, given $\mathbf{b} \in \mathbb{R}^n$.

1b

How many flops are required to compute $\mathbf{x}$?

Problem 2 Cholesky

Find the Cholesky factorization, LDL^T, of the matrix

$$A = \begin{bmatrix} 2 & -4 \\ -4 & 11 \end{bmatrix}.$$

Is A positive definite?

Problem 3 Iterative scheme

Consider the linear system $A\mathbf{x} = \mathbf{b}$ where A is a non-singular $n \times n$ matrix and $\mathbf{b}$ is a vector in $\mathbb{R}^n$. An iterative scheme for solving the system can be

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derived from a splitting matrix B. What is then the iterative scheme and what is the condition on A and B to guarantee convergence?

Problem 4 Polynomial interpolation

The Chebyshev polynomial of degree $n \geq 0$ is

$$T_n(x) := \cos(n \arccos x), \qquad -1 \leq x \leq 1.$$

4a

Derive a formula for T_{n+1} as a function of T_n and T_{n-1}, and use it to find T_2.

4b

What are the zeros of T_2 and why are they suitable for polynomial interpolation?

Problem 5 Bernstein basis

A Bézier curve of degree d in $\mathbb{R}^2$ is a parametric curve

$$\mathbf{p}(t) := \sum_{i=0}^{d} B_{i,d}(t)\mathbf{c}_i, \qquad 0 \leq t \leq 1,$$

with control points $\mathbf{c}_0, \mathbf{c}_1, \ldots, \mathbf{c}_d \in \mathbb{R}^2$, and

$$B_{i,d}(t) := \binom{d}{i} t^i (1-t)^{d-i}.$$

Show that the length of $\mathbf{p}$ is bounded above by the length of its control polygon (the polygon passing through its control points). Hint: you can use the fact that

$$\mathbf{p}'(t) = d \sum_{i=0}^{d-1} B_{i,d-1}(t)(\mathbf{c}_{i+1} - \mathbf{c}_i).$$

Problem 6 Newton's method

6a

Given $f : \mathbb{R} \to \mathbb{R}$, what is Newton's method for finding a root x_* of the equation $f(x_*) = 0$?

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6b

Let

$$f(x) := x^2(x+1), \qquad x \in \mathbb{R}.$$

What is the expected order of convergence to the root $x_* = -1$? Determine the order of convergence to the root $x_* = 0$.

Problem 7 Gradient descent

Suppose A is a symmetric positive definite $n \times n$ matrix and $\mathbf{b} \in \mathbb{R}^n$. The gradient descent method attempts to solve $A\mathbf{x}^* = \mathbf{b}$ by iteratively minimizing

$$F(\mathbf{x}) := \frac{1}{2}\mathbf{x}^T A\mathbf{x} - \mathbf{x}^T\mathbf{b}, \qquad \mathbf{x} \in \mathbb{R}^n.$$

If $\mathbf{x}$ is the current approximation to $\mathbf{x}^*$, we choose a direction $\mathbf{d} \in \mathbb{R}^n$ such that $\mathbf{d}^T\nabla F(\mathbf{x}) < 0$ and the new approximation is

$$\mathbf{x}' = \mathbf{x} - \frac{\mathbf{d}^T\nabla F(\mathbf{x})}{\mathbf{d}^T A\mathbf{d}}\mathbf{d}.$$

Show that the new gradient, $\nabla F(\mathbf{x}')$, is orthogonal to $\mathbf{d}$.

Good luck!