MAT3110-MAT4110, Autumn 2025, Compulsory assignment 2

Deadline 23 October, 14:30

An 8-bit black and white image can be represented as an $n \times m$ matrix, consisting of integer values between 0 and 255 (or normalized to have values between 0 and 1) which define the greyscales of the pixels m_{ij} for all i and j. We will study how the SVD decomposition can be used to compress such an image. First download three example images and convert them to $n \times m$ matrices whose elements have values between 0 and 1.

```
% Convert the images to grey scale
% https://pixabay.com/en/board-chess-chessboard-black-white-157165/
im1 = im2gray(imread("board-157165_1280.png"));
% https://pixabay.com/en/jellyfish-under-water-sea-ocean-698521/
im2 = rgb2gray(imread("jellyfish-698521_1280.jpg"));
% https://pixabay.com/photos/outdoors-new-york-skyline-nyc-city-5129182/
im3 = rgb2gray(imread("outdoors-5129182_1280.jpg"));
% Convert to double between 0 and 1.
im1 = im2double(im1);
im2 = im2double(im2);
im3 = im2double(im3);
% Plot the images
figure
imshow(im1,"InitialMagnification",50)
figure
imshow(im2,"InitialMagnification",50)
figure
imshow(im3,"InitialMagnification",50)
```

Recall that the singular values of an $n \times m$ matrix are $\sigma_1 \geq \sigma_2 \geq \cdots \geq 0$. Your task is to compress the images using the SVD decomposition of the

matrices and by retaining only the first r singular values for some chosen r: a rank r approximation. If

$$\text{compression ratio} = \frac{\text{uncompressed size}}{\text{compressed size}},$$

what is the compression ratio as a function of n, m, and r?

For the three example images above, make a plot of the log of the singular values. Then, for each image, make a compressed image that you think is visually acceptable by choosing some appropriate r.

Your delivery should be a short report summarizing your work as a **single pdf file**, submitted through canvas.