27th February, 2025

MAT3250

Mandatory assignment 1 of 1

Submission deadline

Thursday 13th MARCH 2025, 14:30 in Canvas (canvas.uio.no).

Instructions

Note that you have **one attempt** to pass the assignment. This means that there are no second attempts.

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with LaTeX). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

In exercises where you are asked to write a computer program, you need to hand in the code along with the rest of the assignment. It is important that the submitted program contains a trial run, so that it is easy to see the result of the code.

Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) no later than the same day as the deadline.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

Complete guidelines about delivery of mandatory assignments:

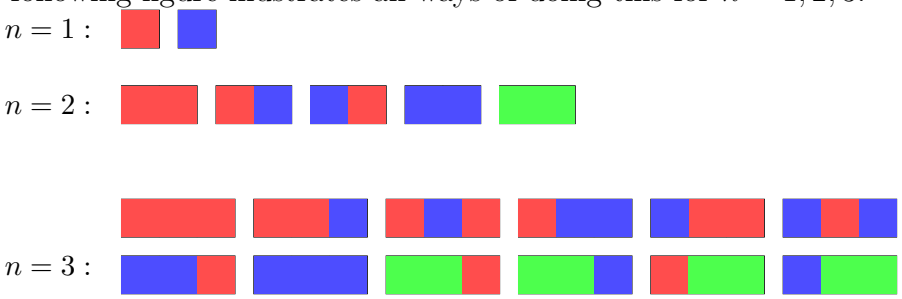
uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

The assignment consists of 9 subproblems, and a minimum of 4 of these must be correctly solved for the assignment to be considered complete. You are encouraged to collaborate on the assignment, but make sure that the write-up of solutions is your own, and that you note the names of any collaborators in your submission. You may answer in Norwegian or English. Good luck!

Problem 1. Consider a $1 \times n$ rectangle which is to be covered by the following three kinds of blocks:

- red 1×1-blocks
- blue 1×1-blocks
- green 1×2-blocks

The following figure illustrates all ways of doing this for $n = 1, 2, 3$:



Let $a_0 = 1$, and for $n \geq 1$, let a_n be the number of coverings as above.

(a) Find a recurrence relation for a_n in terms of a_{n-1} and a_{n-2} for all $n \geq 2$.

(b) Express the generating function of the sequence a_n as a rational function.

(c) Find an integer k and real numbers $b_1, \ldots, b_k$, $c_1, \ldots c_k$ such that

$$a_n = \sum_{i=1}^{k} c_i b_i^n$$

holds for all $n \geq 0$.

(d) For which numbers $b \in \mathbb{R}_{>0}$ is it true that the function $f(n) = a_n$ lies in $O(b^n)$?

Problem 2. What is the number of permutations π of the set $\{1, 2, \ldots, n\}$ such that none of $1, 2$ or 3 are fixed points of π?

Problem 3. Let $G = (V, E)$ be a graph. A circuit of length l in G is a subgraph C of G with l distinct vertices $u_1, u_2, \ldots, u_l$ and l distinct edges $u_1u_2, u_2u_3, \ldots, u_{l-1}u_l, u_lu_1$. A **Hamiltonian circuit** of G is a circuit containing every vertex of G.

(a) Find all Hamiltonian circuits in the hypercube graph Q_3 that contain the edges $(0,0,0)(0,0,1)$ and $(0,0,1)(0,1,1)$.

(b) Find all Hamiltonian circuits in the complete bipartite graph $K_{3,3}$.

(c) Show that $K_{3,4}$ has no Hamiltonian circuit.

(d) For $n \geq 3$, find the number of Hamiltonian circuits in K_n.