UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Examination in:	INF-MAT3360 — Partial Differential Equations
Day of examination:	11. June 2012.
Examination hours:	09.00 – 13.00.

This examination set consists of 3 pages.

Appendices:	None
Permitted aids:	None.

Make sure that your copy of the examination set is complete before you start solving the problems.

All answers must be justified. Yes or no does not count as answers. 20p are possible. Most exercises give 1p, but some give 2p.

Problem 1.

Consider the following heat equation:

$$\begin{cases} u_t - u_{xx} = 0 & \text{for all} \quad (x,t) \in (0,1) \times (0,T] \\ u(0,t) = u(1,t) = 0 & \text{for all} \quad t \in (0,T] \\ u(x,0) = f(x) & \text{for all} \quad x \in [0,1] \end{cases} \tag{1}$$

(i) Prove uniqueness. (2p)

(ii) Write up an explicit and an implicit scheme. (1p)

(iii) Find suitable stability requirements for both schemes by using von Neumann analysis. (2p)

(Continued on page 2.)

Problem 2.

Consider the following two point boundary value problem:

$$\begin{cases} -u'' + u = f & \text{for all } x \in (0,1), \\ u(0) = u(1) = 0, \end{cases} \tag{2}$$

and we denote the differential operator as L, i.e., $Lu = -u'' + u$.

(i) Is the differential operator L positive (for C_0^2 functions)? (1p)

(ii) Is L symmetric (for C_0^2 functions)? (1p)

(iii) Is L linear (for C_0^2 functions)? (1p)

(iv) Is there a unique solution for this boundary value problem? (1p)

(v) Write down a difference scheme that solves the problem numerically. (1p)

(vi) Does the scheme produce a symmetric discrete operator? (1p)

(vii) Is the scheme explicit or implicit? (1p)

Problem 3.

Consider the following heat equation:

$$\begin{cases} u_t - u_{xx} + u(1-u) = 0 & \text{for all} \quad (x,t) \in (0,1) \times (0,T] \\ u(0,t) = u(1,t) = 0 & \text{for all} \quad t \in (0,T] \\ u(x,0) = f(x) & \text{for all} \quad x \in [0,1] \end{cases} \tag{3}$$

(i) Is the problem linear? (1p)

(ii) Write up an explicit and an implicit scheme. (1p)

(iii) Find invariant regions and a stability requirement for the explicit scheme. (2p)

(Continued on page 3.)

Problem 4.

Consider the following boundary value problem:

$$
\begin{aligned}
-\Delta u + u &= f && \text{for all } x, y \text{ in } (0,1), && (4)\\
u_y(x,0) &= 0 && \text{for all } x \text{ in } (0,1), && (5)\\
u_y(x,1) &= 0 && \text{for all } x \text{ in } (0,1), && (6)\\
u_x(0,y) &= 0 && \text{for all } y \text{ in } (0,1), && (7)\\
u_x(1,y) &= 0 && \text{for all } y \text{ in } (0,1), && (8)\\
& && && (9)
\end{aligned}
$$

where $-\Delta u = -\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2}$. We denote the differential operator by L, i.e., $Lu = u - \Delta u$.

(i) Is the differential operator L positive for C^2 functions? (2p)

(ii) Is it symmetric for C^2 functions? (1p)

(iii) Is it linear for C^2 functions? (1p)

END