UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Examination in:	INF-MAT3360 — Partial Differential Equations
Day of examination:	21. March 2013.
Examination hours:	09.00 – 13.00.
This examination set consists of 2 pages.	
Appendices:	None
Permitted aids:	None.

Make sure that your copy of the examination set is complete before you start solving the problems.

All answers must be justified. Yes or no does not count as answers.

Problem 1.

Consider the following two point boundary value problem:

$$\begin{cases} Lu = -cu'' + 42u = f \quad \text{for all } x \in (0,1), \\ u(0) = u(1) = 0, \end{cases} \tag{1}$$

Here, $c \in \mathbb{R}$.

(i) Is L symmetric? Is L linear?

(ii) For which c is L positive? Is there a unique solution?

(ii) What are the eigenvalues and eigenvectors ?

(iii) Find a formula for the solution by using the metod of Fourier.

(iv) Write down a difference scheme that solves the problem numerically. Is the scheme explicit or implicit?

(Continued on page 2.)

Problem 2.

Consider the following system of ODEs:

$$\begin{aligned}
S' &= \Sigma - \beta R - \delta S, \\
I' &= \beta R - \rho I - \delta I, \\
Z' &= \rho I + \zeta R \\
R' &= \delta S + \delta I - \zeta R.
\end{aligned}$$

(i) Write an explict and an implicit scheme for the solution of this ODE system.

(ii) What is the order of the equations? Is the system linear? Show how to determined whether the system is linear or not.

(iii) Write code (e.g., Matlab, Python, or C) to solve the system.

Problem 3.

Consider the following heat equation:

$$\begin{cases} u_t - u_{xx} + cu = 0 & \text{for all} \quad (x,t) \in (0,1) \times (0,T] \\ u(0,t) = u(1,t) = 0 & \text{for all} \quad t \in (0,T] \\ u(x,0) = f(x) & \text{for all} \quad x \in [0,1] \end{cases} \tag{2}$$

Here, $c > 0$.

(i) Let u_1 and u_2 be two solutions with different initial conditions, f_1 og f_2. How large may the difference between u_1 and u_2 be for $t = T$?

(ii) Write up an explicit and an implicit scheme.

(iii) Find suitable stability requirements for both schemes by using von Neumann analysis.

(iv) Show that the energy decreases with time.

END