

# Curriculum MAT3360 Spring 2023

The following sections in Tveito and Winther's book *Introduction to partial differential equations* are the curriculum:

**1.1-1.4**

**2.1-2.4 (excluding 2.2.3)**

**3.1-3.8**

**4.1-4.5**

**5.1-5.3**

**6.1-6.4**

**7.1-7.5**

**8.1-8.5**

**9.1-9.5**

**10.1-10.2.**

## Summary of important topics:

### Chapter 1

- 1.1-1.3 Terminology for differential equations (linear vs nonlinear, PDE vs ODE, order, homogeneous vs nonhomogeneous).
- 1.4 method of characteristics for first-order linear PDE, explicit solutions of heat equation and wave equation.

## Chapter 2

- 2.1 Poisson's equation on  $(0, 1)$ , explicit solution derived by a Green's function. Uniqueness and boundedness of solution.
- 2.2-2.3 Finite difference method for solving Poisson's equation. Properties of the operator  $Lu = -u''$  and the related discrete operator  $L_h$ . Existence, uniqueness, max principle, convergence rate for discrete solution.
- 2.4 Eigenvalue problem for Poisson operator  $L$  on  $C_0^2((0, 1))$  and for discrete operator  $L_h$  on  $D_{h,0}$ .

## Chapter 3

- 3.1-3.4 Particular solutions of heat equation on  $x \in (0, 1), t > 0$  with zero-valued Dirichlet boundary conditions using separation of variables. Formal solution by principle of superposition.
- 3.5-3.6 Particular solutions and formal solution of heat equation with zero-value Neumann boundary conditions (and some extensions to other BC, like Dirichlet non-zero and periodic in exercises).
- 3.7-3.8 Energy arguments for heat equation. Uniqueness of solutions.

## Chapter 4

- 4.1-4.3 Explicit finite difference scheme for heat equation, von Neumann stability analysis. Use case: derive growth condition for numerical scheme on size of  $\Delta t$  relative to  $\Delta x$  for a given PDE.
- 4.4 Implicit finite difference scheme for heat equation, stability analysis.
- 4.5 Energy arguments for numerical methods.

## Chapter 5

- 5.1-5.2 Separation of variables for wave equation on  $x \in (0, 1)$ . Formal solutions adapting Fourier coefficients to initial conditions. Energy argument, uniqueness of solution.
- 5.3 Explicit finite difference method for solving wave equation, von Neumann stability analysis.

## Chapter 6

Maximum principles for:

- 6.1 two-point boundary value problems
- 6.2-6.3 (excluding 6.3.2) linear heat equation and for numerical solutions of linear heat eq. And for nonlinear heat equations.
- 6.4 Poisson's equation in 2D and for numerical solutions of this problem. Theory on harmonic and discrete harmonic functions.

## Chapter 7

Topics on Poisson's equation in 2D:

- 7.1-7.2 Formal solutions in square and circular domains, polar coordinates.
- 7.3-7.4 Divergence theorem, symmetry and positive definiteness of the negative Laplace operator on  $C_0^2(\Omega)$ . Mean value property for harmonic functions in 2D.
- 7.5 properties of the discrete negative Laplace operator  $L_h$  on  $D_{h,0}$ . Existence and uniqueness of numerical solution, and convergence rate.

## Chapter 8

- 8.1 Fourier series (for functions) on  $[-1, 1]$ , relation to Sine and Cosine series through even and odd functions. Differentiation of Fourier series.
- 8.2 General Fourier series and Sturm–Liouville problems.
- 8.3-8.5 Mean square distance, Cauchy–Schwarz inequality, Bessel’s inequality, Parseval’s identity and its relation to convergence of Fourier series in mean square sense. Poincare’s inequality.

## Chapter 9

- 9.1 Notions of convergence for sequences of functions: mean square sense, pointwise and uniform.  
Nice property of uniform convergence: regularity of sequence of functions carries over to the limit function that the sequence converges to.
- 9.2-9.4 Pointwise, uniform and mean square sense convergence of Fourier series on  $[-1, 1]$ : sufficient regularity conditions on function  $f$  for the respective convergences to hold.
- 9.5 Relation between smoothness of function  $f$  and rate of decay of its Fourier coefficients.

## Chapter 10

- 10.1 Show equivalence between uniform convergence of Fourier series on  $[-1, 1]$  and Sine series on  $[0, 1]$  when restricted to particular to function space  $C_{p,0}^m$  on  $[0, 1]$  and odd extension of such functions on  $[-1, 1]$ .
- 10.2 Studying heat equation on  $u_t = u_{xx}$  on  $x \in (0, 1), t > 0$  mainly with  $u(x, 0) = f$  for some  $f \in C_{p,0}^m$ .  
Show that the formal solution – the limit of linear combinations of  $N$  particular solutions as  $N \rightarrow \infty$  – is indeed a true solution of the heat equation. Show how regularity of the formal solution (which we by now know is the unique solution) depends on the regularity of  $f$ .