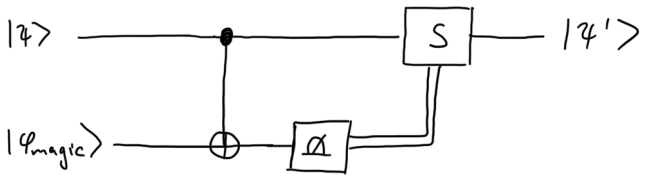
EXERCISES 7

Excercise 1 (Gate teleportation)**.** Consider the so-called magic state $|\phi_{\text{magic}}\rangle \in \mathbb{C}^2$ given by

$$|\phi_{\text{magic}}\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\pi/4}|1\rangle\right).$$

Consider the following circuit: Here, the state $|\psi\rangle \in \mathbb{C}^2$ is some unknown input



state and the measurement is with respect to the computational basis. Show that $|\psi'\rangle = T|\psi\rangle$, i.e., the circuit implements a T-gate using the magic state as a resource.

Excercise 2. Consider a qubit unitary U with real entries in the computational basis. Show that there are signs $s_1, s_2, s_3 \in \{\pm 1\}$ and $\theta \in [0, 2\pi)$ such that

$$U = s_1 \begin{pmatrix} 1 & 0 \\ 0 & s_2 \end{pmatrix} \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & s_3 \end{pmatrix}.$$

Excercise 3. Consider the operator norm of a linear operator $M : \mathbb{C}^n \to \mathbb{C}^n$ defined as

$$\|M\| = \max_{|\psi\rangle \in \mathbb{C}^n, \langle\psi|\psi\rangle = 1} \|M|\psi\rangle\|.$$

Prove the following properties of the operator norm:

(1) The norm $\|M\|$ equals the largest singular value of M.
(2) The operator norm is submultiplicative, i.e., $\|M_1 M_2\| \leqslant \|M_1\|\|M_2\|$ for operators $M_1, M_2 : \mathbb{C}^n \to \mathbb{C}^n$.
(3) For operators $M_1 : \mathbb{C}^n \to \mathbb{C}^n$ and $M_2 : \mathbb{C}^m \to \mathbb{C}^m$ we have $\|M_1 \otimes M_2\| = \|M_1\|\|M_2\|$.
(4) If $U : \mathbb{C}^n \to \mathbb{C}^n$ is unitary, then we have $\|U\| = 1$.