

# MAT3440 Spring 2025 – curriculum and exam relevant topics

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## Curriculum - HSD

See lecture notes for the sections below that only have been covered partly.

- HSD: 1.1-5, 2.1-7, 3.1-3.4, 4.1-2, 6.1-5, 7.1-7.4, 8.1-5, 9.1-4, 10.1 and Poincare-Bendixson theorem (“easier” version in the lecture notes than in the book), parts of 11.1-11.2, 13.1-3, 13.6 and parts of 13.4, and 17.1-5 and parts of 17.6.
- Lecture notes on numerical methods for ODEs by U. Fjordholm: Chapters 1-4.

## Exam relevant topics

- General theory for  $X' = F(X, t)$ :
  - General solution techniques: separation of variables and integrating factor for 1D equations
  - Picard–Lindelöf theorem for local existence and uniqueness, the maximal solution interval for an ODE. When can one apply the theorem.
  - Understand by some special cases when a problem may have a bounded maximal solution interval and when a problem may have more than one solution (non-uniqueness).
- Linear systems  $X' = AX + g(t)$ :
  - For planar systems  $X' = AX$ ; find general solution for any matrix  $A \in \mathbb{R}^{2 \times 2}$ , draw phase portrait and classify the EP 0
  - In general dimension  $n \geq 1$ ; given the eigenpairs of a diagonalizable matrix  $A$ , express general solution of ODE
  - Properties covered on matrix exponentials, how to compute  $e^A$  in simple settings, and how it relates to solutions of linear systems of ODE
  - Solution formula for inhomogeneous problem  $X' = AX + g(t)$
- Autonomous systems  $X' = F(X)$ :
  - Definitions for classifying properties equilibrium points
  - Identify EPs and draw phase lines/phase portraits for 1D/2D problems
  - Use the linearization theorem to classify EPs
  - Level sets and Lyapunov functions/theory (also to classify EPs)

- Special case: Lyapunov theory for gradient systems  $X' = -\nabla V(X)$
  - Hamiltonian systems: representation and conserved quantities
  - Rewrite system in polar coordinates for 2D problems
  - Know definitions of nullclines and how to use them to find positively invariant sets
  - Know definition of  $\alpha(X)$  and  $\omega(X)$  limit sets, Poincaré–Bendixson theorem for verifying the existence of periodic solutions/closed orbits
  - Definitions: closed orbits and limit cycles.
- Bifurcation theory for  $X' = F_\beta(X)$  for a parameter  $\beta \in \mathbb{R}$  for 1D and 2D problems:
    - Identify bifurcation points
    - Draw bifurcation diagram
    - Example problem family:  $X' = A(\beta)X$  where  $A(\beta) \in \mathbb{R}^{2 \times 2}$
- Numerical methods for  $X' = F(X, t)$ :
    - Will restrict ourselves to one-stage or two-stage Runge–Kutta methods for all bullets below
    - Understand Butcher tableau representation of Runge–Kutta methods
    - Compute a few iterations of explicit and implicit Runge–Kutta methods
    - Derive the stability function and stability region of a method
    - Definitions: order, local error and absolutely stable,
    - For 2D problems  $X' = AX$  with negative-definite, symmetric  $A \in \mathbb{R}^{2 \times 2}$ , be able to determine an upper bound for the timestep such that  $|\bar{X}_n|$  is bounded for all  $n \geq 0$  and any  $X_0 \in \mathbb{R}^2$ .