UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: MAT3500/4500 — Topology

Day of examination: Friday 16 December 2011

Examination hours: 09.00 – 13.00

This problem set consists of 2 pages.

Appendices: None

Permitted aids: None

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1

1a

Let (X, d) be a metric space. If $A \subset X$ is a subspace and x a point in X, the distance $d(x, A)$ from x to A is defined by the formula

$$d(x, A) = \inf\{d(x, a) \mid a \in A\}.$$

Show that $d(x, A) = 0$ if and only if $x \in \overline{A}$. (The closure of A in X.)

1b

Define what it means that space is *regular*.
Show that every metric space is regular.

1c

Prove that a topological space X is Hausdorff if and only if

$$\bigcap_{x \in V,\ V \text{ open}} \overline{V} = \{x\}$$

for every $x \in X$.

Problem 2

2a

Define what is means that a space X is *locally compact*.

If the space X is locally compact and Hausdorff, what is the topology on the one-point compactification $X^* = X \cup \{P_X\}$?

(Continued on page 2.)

2b

A continuous map $f : X \to Y$ is called *proper* if $f^{-1}(K)$ is compact for every compact $K \subset Y$.

Show that if X is compact and Y is Hausdorff, then every continuous map $f : X \to Y$ is proper.

2c

Now assume that X and Y are locally compact Hausdorff, and let $f^* : X^* \to Y^*$ be defined by $f^*(x) = f(x)$ for $x \in X$ and $f^*(P_X) = P_Y$. Show that f^* is continuous if and only if f is proper.

Problem 3

In this problem $p : E \to B$ is a covering map between path connected spaces.

3a

Show that p is an open map.

3b

Assume that the fundamental group of B has exactly two elements. Show that either E is simply connected or p is a homeomorphism, but not both.

END