UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: MAT3500/4500 — Topology

Day of examination: Tuesday December 18th 2012

Examination hours: 09.00 – 13.00

This problem set consists of 2 pages.

Appendices: None

Permitted aids: None

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Each subproblem (1a, 1b, ..., 4b) carries the same weight. You may use the results of an earlier subproblem to answer later questions, even if you have not answered the earlier one.

Problem 1

Let $\mathcal{T}$ be the collection of subsets of $\mathbb{R}$ that contain the integers $\mathbb{Z}$ or are empty, i.e.

$$\mathcal{T} = \{U \subset \mathbb{R} \mid U = \emptyset \text{ or } \mathbb{Z} \subset U\}$$

1a

Show that $\mathcal{T}$ is a topology.

We now consider $\mathbb{R}$ with this topology $\mathcal{T}$.

1b

Show that $K \subset \mathbb{R}$ is compact if and only if K only contains a finite number of nonintegers.

1c

Determine all connected subsets of $\mathbb{R}$.

Problem 2

2a

Give the definition of a locally compact topological space.

(Continued on page 2.)

2b

Let X be a compact Hausdorff space, $x \in X$ and $Y = X \setminus \{x\}$ with the subspace topology. Let Y^* be the one-point compactification of Y. Show that Y^* is homeomorphic to X.

2c

Let $S_n \subset \mathbb{R}^2$ be the circle or radius $1/n$ and center $(1/n, 0)$, X the compact set

$$X = \bigcup_{n=1}^{\infty} S_n$$

and $Z \subset \mathbb{R}^2$ the subset consisting of all lines $\{n\} \times \mathbb{R}$ for $n = 1, 2, \cdots$. Show that the one-point compactification Z^* is homeomorphic to X.

Problem 3

Let $p : E \to B$ be a covering map such that $p^{-1}(b)$ is finite for all $b \in B$.

3a

Give an example where E is a path connected compact Hausdorff space and p is not a homeomorphism. (You can either give an explicit covering map or provide an appropriate figure.)

3b

Is it possible to find an example as in a) with a simply connected B ? Explain your answer.

Problem 4

In this problem $f : X \to Y$ is a quotient map. The *saturation* of a set $A \subset X$ is the set $f^{-1}(f(A))$.

4a

Show that f is a closed map if and only if the saturation of any closed set A also is closed.

4b

Show that if f is a closed map and X a normal space, then so is Y.

END