19th October, 2017

MAT 4200

Mandatory assignment 1 of 1

Submission deadline

Thursday 9th November 2017, 14:30 at Devilry (https://devilry.ifi.uio.no).

Instructions

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with LaTeX). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. Students who fail the assignment, but have made a genuine effort at solving the exercises, are given a second attempt at revising their answers. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

In exercises where you are asked to write a computer program, you need to hand in the code along with the rest of the assignment. It is important that the submitted program contains a trial run, so that it is easy to see the result of the code.

Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) well before the deadline.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

Exercise 1

Let R be a Noetherian ring, $\mathbf{p} \subset R$ a prime ideal and M a finitely generated R-module. The prime ideal $\mathbf{p}$ is associated to M if there exists $x \in M$, $x \neq 0$ such that $Ann(x) = \mathbf{p}$, and we write $\mathbf{p} \in Ass(M)$.

a) Show that $Ass(M) = \emptyset$ if and only if $M = 0$.

b) Show that for a prime ideal $\mathbf{p} \subset R$, we have $Ass(R/\mathbf{p}) = \{\mathbf{p}\}$

c) Let $\mathbf{p} \in Ass(M)$. Show that there exists a submodule of M which is isomorphic to $R/\mathbf{p}$.

d) Let $0 \to M' \to M \to M'' \to 0$ be a short-exact sequence of R-modules. Then

$$Ass(M) \subseteq Ass(M') \cup Ass(M'')$$

e) Let $S \subset R$ be a multiplicative subset. Show that $\mathbf{p} \mapsto S^{-1}\mathbf{p}$ gives a bijection

$$\{\mathbf{p} \in Ass(R) \,|\, \mathbf{p} \cap S = \emptyset\} \to Ass(S^{-1}R)$$

Exercise 2

Let R be a ring and M an R-module. The set of prime ideals $\mathbf{p} \subset R$ such that $M_{\mathbf{p}} \neq 0$ is called the **support** of M, denoted $Supp(M)$. We use the notation $V(I)$ for the set of prime ideals, containing I; $V(I) = \{\mathbf{p} \subset R \,|\, I \subseteq \mathbf{p},\ \mathbf{p}\,\text{prime ideal}\}$.

a) Show that $Supp(R/I) = V(I)$.

b) Let M be a finitely generated R-module. Show that $Supp(M) = V(Ann(M))$.

c) Let $0 \to M' \to M \to M'' \to 0$ be a short-exact sequence of R-modules. Then

$$Supp(M) = Supp(M') \cup Supp(M'')$$

Exercise 3

Let M be a finitely generated modules over a Noetherian ring R, and let $\mathbf{p}$ be a prime ideal of R. In this exercise you can use the equality

$$\bigcap_{\mathbf{p} \in Ass(M)} \mathbf{p} = \mathcal{R}(Ann(M))$$

without any proof.

a) Show that $Ass(M) \subseteq Supp(M)$.

b) Show that $\mathbf{p} \in Supp(M)$if and only if there is a prime ideal $\mathbf{p}' \in Ass(M)$, such that $\mathbf{p}' \subseteq \mathbf{p}$

c) Use a) and b) to show that the minimal elements of $Supp(M)$ are the same as the minimal elements of $Ass(M)$.

Exercise 4

a) Find $Ass(M)$ and $Supp(M)$ for the $\mathbb{Z}$-module $M = \mathbb{Z}/(60)$.

b) Find $Ass(M)$ and $Supp(M)$ for the $R = k[x, y, z]$-module $M = R/(xyz, x^2z)$.