From [E]: 11.1

From [AM]: 5.5 i), 5.6., 5.7.

Hint for 5.7.: Assume $b \in B \setminus A$ is a root of $f = x^n + a_{n-1}x^{n-1} + \cdots + a_0$. Write $f = (x - b)g$ and examine the coefficients of g.

(1) Let A be an integral domain, let $K = K(A)$, and let $\overline{K}$ be an algebraic closure of A. Prove that $x \in \overline{K}$ is integral over A if and only if the unique monic irreducible polynomial of x in $K[x]$ has coefficients in A.

(2) Let $A \subseteq B$ be rings, and let $b \in B$. Prove that the set $\{a \in A \mid ab \text{ is integral over } A\}$ is an ideal of A.