Suggested problems

From [E]: 8.1, 8.3, 8.4, 8.23, 8.25
From [AM]: 6.1., 6.2., 6.3, 6.4.

(1) Let k be a field, and consider

$$k[x]_x = \left\{ \frac{f}{x^n} \mid n \geq 0, f \in k[x] \right\} \subseteq k(x)$$

as a $k[x]$-module, along with the submodule $k[x] \subseteq k[x]_x$. Prove that (as $k[x]$-modules)

- $k[x]_x$ is not Noetherian
- $k[x]_x$ is not Artinian
- $k[x]_x/k[x]$ is not Noetherian
- $k[x]_x/k[x]$ is Artinian (*)

For the final point, let $M \subseteq k[x]_x/k[x]$ be a $k[x]$-submodule and prove that

$$\frac{x^n + \cdots + a_0}{x^m} \in M \Leftrightarrow x^{n-m} \in M.$$

Use this to prove that if $0 \subsetneq M \subsetneq k[x]_x/k[x]$, then there is some n such that

$$M = \left\{ \frac{f}{x^n} \mid f \in k[x] \right\}.$$