MAT4210 - Algebraic Geometry 1

Mandatory assignment

Deadline

Thursday 27$^{\text{th}}$ February 2020, 12:00 in Canvas (canvas.uio.no) or handed-in during the lecture.

Instructions

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with LaTeX). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. Students who fail the assignment, but have made a genuine effort at solving the exercises, are given a second attempt at revising their answers. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

In exercises where you are asked to write a computer program, you need to hand in the code along with the rest of the assignment. It is important that the submitted program contains a trial run, so that it is easy to see the result of the code.

Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) well before the deadline.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

Problem 1. Let $X = Z(x^3 - xy^3) \subset \mathbb{A}^2$. Determine the irreducible components and compute their dimension.

Problem 2. Let $X = Z(x^2 - y^2 - x^6 - y^6) \subset \mathbb{A}^2$. Determine the dimension of X, study the non-singularity in the origin, determine the multuplicity of X at the origin and its tangent directions (in particular, prove that the origin is a node).

Bonus question: Let $\phi\colon \text{Bl}_0\, \mathbb{A}^2 \to \mathbb{A}^2$ be the blow up and $\varphi\colon \widetilde{X} \to X$ its restriction to the strict transform $\widetilde{X}$ of X. Show that $\widetilde{X}$ is non-singular at all points $p \in \varphi^{-1}(0)$.