

Curriculum MAT4301 2025 from LC Evans "Partial differential equations" 2nd ed

Pages 1-2 list **all** of the topics that are **relevant for the exam**.

On pages 3-4, we provide the full curriculum of the course. This is a longer list of all the topics we have covered in the course.

Exam relevant topics

The term **working knowledge** used below means to be able to use theorems and formulas as tools to compute solutions and properties of solutions.

- Appendices:
 - A: Integral averages of functions over spheres and balls, notation for derivatives of functions and function spaces, oh-notation (i.e., $\mathcal{O}(h)$ and $o(h)$).
 - B: Young's inequality and Cauchy–Schwarz inequality.
 - C: Working knowledge of C.2 integration by parts, Gauss–Green theorem and Green's formulas, C.3 polar coordinates, and C.4 the standard mollifier and mollification of functions.
- §2.1: Transport equation. Solve initial-value problem transport equations and related nonhomogeneous PDE with the method of characteristics.
- §2.2: Laplace equation
 - 2.2.1-2: Derivation and representation of the fundamental solution. Using the fundamental solution to solve the Poisson equation on $\mathbb{R}^n$ and being able to prove that it is a solution. Being able to prove the mean-value formulas.
 - 2.2.3: Use mean-value formulas to prove the strong maximum principle. Use the strong maximum principle to prove uniqueness for the Poisson equation.
 - 2.2.4.a: Working knowledge of how a Green's function can be used to solve boundary-value problems.
 - 2.2.5: Energy methods for Poisson's equation on bounded domains.
- §2.3: Heat equation
 - 2.3.1.a-b: How to solve initial value problems using the fundamental solution.
 - 2.3.1.c: Solving the nonhomogeneous problems on the global domain using Duhamel's principle.
 - 2.3.3.: Working knowledge of the mean-value formula, strong maximum principle and uniqueness for the heat equation on bounded domains. Extension to similar properties on unbounded domains when restricting to a class of bounded solutions.
 - 2.3.4: Energy methods for heat equations on parabolic cylinders (for the forward-in-time problem).
- §2.4: Wave equation
 - 2.4.1.a: Solution of the wave equation in dimension $n = 1$ with d'Alembert's formula. Fundamental properties of the solution for dimension $n = 1$: uniqueness and finite propagation speed.
 - 2.4.2: Duhamel's principle for solving nonhomogeneous wave equations in dimension $n = 1$.
 - 2.4.3: Working knowledge of energy methods for uniqueness for initial/boundary-value wave equation on bounded domains $U \times (0, T]$.

- §3: Nonlinear first-order PDEs
 - §3.1: Not exam relevant.
 - §3.2: The method of characteristics (MoC) for local solutions of first-order PDEs. This includes treatment of boundary conditions, solution of ODEs and local inversion. And being able, with hints, to derive the system of ODEs for the MoC.
 - §3.3: Hamilton–Jacobi equations. Hamilton’s equations, working knowledge of the Legendre transform and how to use the Hopf–Lax formula to solve HJ equations.
 - §3.4: Conservation Laws. Definition of integral solutions, the Rankine–Hugoniot condition, and the definition of entropy solutions. Being able to use the Lax–Oleinik formula to find solutions of simple conservation laws.
- §10: Hamilton–Jacobi equations
 - §10.1-2: Definition of viscosity solutions. Being able to verify that a function is a viscosity solution in simple settings.
 - §10.3: Definition of the value function of an optimal control problem. Working knowledge of how Theorem 10.3.2 connects the unique viscosity solution of a Hamilton–Jacobi–Bellman equation to an associated value function for a control problem.

Full Curriculum of MAT4301

- Appendices:
 - * A: integral averages of functions over spheres and balls, notation for derivatives and function spaces, oh-notation.
 - * B: Young’s inequality and Cauchy–Schwarz inequality.
 - * C:
 - C.1 boundaries (used when studying the method of characteristics, in particular).
 - C.2: Theorem C.2.1 (Gauss–Green), Theorem C.2.2 (Integration by parts), Theorem C.2.3 (Green’s formula).
 - C.3: Polar coordinates formulas and the co-area formula.
 - C.4: Convolution and smoothing. The standard mollifier, mollifications of a function f , and properties of mollifiers (we covered Theorem C.5.7 properties (i) and (iii)).
- §2.1: Transport equation. Solve initial-value problem transport equations and related nonhomogeneous PDE with the method of characteristics.
- §2.2: Laplace equation
 - * 2.2.1: Derivation and representation of the fundamental solution. Using the fundamental solution to solve the Poisson equation on $\mathbb{R}^n$
 - * 2.2.2: Mean-value formulas for the Laplace equation. Theorems 2.2.2 and 2.2.3
 - * 2.2.3: We covered all results in this section. I list the most relevant ones. Theorem 2.2.4 (strong maximum principle) on bounded open domains U . Theorem 2.2.5: Uniqueness for the Poisson equation with Dirichlet BC. Theorem 2.2.6: Smoothness of functions that satisfy the mean-value property. Theorem 2.2.10: Analyticity of harmonic functions.
 - * 2.2.4.a: Green’s function approach to solving boundary-value problems. Theorem 2.2.12: Representation formula.
 - * 2.2.4.b: Green’s function on the half space $\mathbb{R}_+^n$ and Theorem 2.2.14: Poisson’s formula for half-space.
 - * 2.2.5: Energy methods, Theorem 2.5.16: Uniqueness and Theorem 2.5.17: Dirichlet’s principle.
- §2.3: Heat Equation
 - * 2.3.1.a-b: Properties of the fundamental solution and Theorem 2.3.1: usage to solve initial-value problems.
 - * 2.3.1.c: Solving the nonhomogeneous problems on the global domain using Duhamel’s principle, Theorem 2.3.2.
 - * 2.3.2: Heat equations on (bounded) parabolic-cylinder domains $U_T = U \times (0, T]$. Theorem 2.3.3: Mean-value formula.
 - * 2.3.3.: Theorem 2.3.4: Strong maximum principle. Theorem 2.3.5: Uniqueness on bounded domains. Theorem 2.3.6-7: Maximum principle and uniqueness on unbounded domains by imposing further boundedness assumptions on the solutions.
 - * 2.3.4: Energy methods. Theorem 2.3.10: Uniqueness of the solution to the forward problem on the parabolic-cylinder. Theorem 2.3.11: Uniqueness of the solution to the backward problem on the parabolic cylinder.

- §2.4: Wave equation
 - * 2.4.1.a: Theorem 2.4.1: Solution of problem in dimension $n = 1$ with d'Alembert's formula.
 - * 2.4.1.b-c: Using the Euler–Poisson–Darboux equation to derive solution formulas for problems in dimension $n = 3$, the Kirchhoff formula, and $n = 2$, the Poisson formula. Huygen's principle. Understand how these three solution formulas for $n = 1, 2, 3$ show uniqueness for the wave equation.
 - * 2.4.2: Using Duhamel's principle to solve nonhomogeneous wave equations for $n \leq 3$.
 - * 2.4.3: Energy methods. Theorem 2.3.5: Uniqueness for initial/boundary-value wave equation on bounded domains $U \times (0, T]$. Theorem 2.3.6: Finite propagation speed.
- §3: Nonlinear first-order PDEs
 - §3.1: Definition and application of complete and general integrals.
 - §3.2: Method of characteristics (MoC). Theorem 3.2.1: Derivation/structure of the characteristic ODE. Section 3.2.3 Treatment of boundary conditions and Theorem 3.2.2: Local existence of solutions. Applications of the MoC to solve PDE (locally or globally).
 - §3.3: Hamilton–Jacobi equations. 3.3.1 Calculus of variations and Theorem 3.3.1: The Euler–Lagrange equation. Theorem 3.3.2: Hamilton's equations. Understanding the Legendre transform. The Hopf–Lax formula for weak solutions of the Hamilton–Jacobi equation. Section 3.3.3: We gave an overview without proofs of existence and uniqueness of weak solutions, Theorem 3.3.8 is the main result.
 - §3.4: Conservation Laws.
 - * §3.4.1: Definition of integral solutions, formation of shocks, derivation of the Rankine–Hugoniot condition.
 - * §3.4.2: Derivation of the Lax–Oleinik formula in Theorem 3.2.1.
 - * §3.4.3: Definition of entropy solutions, existence of such solutions by Lax–Oleinik formula, and (without proof) Theorem 3.4.3: Uniqueness of entropy solutions.
- §10: Hamilton–Jacobi equations
 - * §10.1-2: Definition of viscosity solutions, and Theorems 10.1-2: Consistency and uniqueness of viscosity solutions.
 - * §10.3: Control theory and dynamic programming. Definition and Theorem 10.3.1: regularity properties of the value function. Theorem 10.3.2: The value function is the unique viscosity solution of the Hamilton–Jacobi–Bellman equation.