Last time

Basic Ideas of Lebesgue Integration

1. Would like to be able to integrate more than just continuous functions.
2. Need to define a notion of "length" for subsets of $\mathbb{R}$ of the form $f^{-1}(O)$, where O is an open set and f is one of the functions we aim to integrate.

Theorem

There exists a smallest collection of functions from $\mathbb{R}$ to $\mathbb{R}$, $\hat{C}$, that contains the continuous functions and is closed under pointwise limits We call $\hat{C}$ the Borel measurable functions.

Theorem

The Borel Measureable functions form an algebra.

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1. Show that $\hat{\mathcal{C}}$ is closed under sup and inf.
2. Introduce the concept of *algebras of sets* and *σ-algebras*. These are meant to be collections of sets that it is reasonable to measure.
3. Define the *Borel sets* as

$$\mathcal{B} = \{B \subseteq \mathbb{R} \mid \chi_B \in \hat{\mathcal{C}}\}.$$

4. Show that $\mathcal{B}$ is a σ-algebra.
5. Show that

$$\hat{\mathcal{C}} = \{f : \mathbb{R} \to \mathbb{R} \mid f^{-1}(O) \in \mathcal{B} \text{ for all open sets } O \subseteq \mathbb{R}\}.$$

6. Show that $\mathcal{B}$ is the smallest σ-algebra containing all the open subsets of $\mathbb{R}$.

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