Last time

1. Introduced the notion of a measure on a σ-algebra.
2. Defined the *Lebesgue outer measure* of any subset $A \subseteq \mathbb{R}$ as

$$\lambda^*(A) = \inf \left\{ \sum_n \ell(I_n) \,\middle|\, \{I_n\}_n \text{ open intervals, } \bigcup_n I_n \supset A \right\}.$$

3. Showed basic properties of λ^*:
   3.1 $A \subseteq B \implies \lambda^*(A) \leq \lambda^*(B)$.
   3.2 $\lambda^* \left(\bigcup_n A_n\right) \leq \sum_n \lambda^*(A_n)$.
   3.3 $\lambda^*(I) = \ell(I)$, for all intervals $I \subseteq \mathbb{R}$.
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1. Study finite additivity of λ^* on $\mathcal{P}(\mathbb{R})$.
   1.1 If $d(A, B) > 0$ then
   $$\lambda^*(A \cup B) = \lambda^*(A) + \lambda^*(B).$$
   1.2 If there is some $a \in \mathbb{R}$ such that $A \subseteq (a, \infty)$ and $B \subseteq (a, \infty)^c$ then
   $$\lambda^*(A \cup B) = \lambda^*(A) + \lambda^*(B).$$
2. Introduce the *Carathéodory criterion*: E satisfies CC if for all W
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