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1. We introduced the concepts of *algebras of sets* and σ-*algebras*. These are meant to be collections of sets that it is reasonable to measure.
2. Defined the *Borel sets* as
$$\mathcal{B} = \{B \subseteq \mathbb{R} \mid \chi_B \in \hat{\mathcal{C}}\}.$$
3. Showed that $\mathcal{B}$ is a σ-algebra.
4. Showed that
$$\hat{\mathcal{C}} = \{f : \mathbb{R} \to \mathbb{R} \mid f^{-1}(O) \in \mathcal{B} \text{ for all open sets } O \subseteq \mathbb{R}\}.$$
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2. Define the *Lebesgue outer measure* of any subset $A \subseteq \mathbb{R}$ as

$$\lambda^*(A) = \inf \left\{ \sum_n \ell(I_n) \,\middle|\, \{I_n\}_n \text{ open intervals, } \bigcup_n I_n \supset A \right\}.$$

3. Show basic properties of λ^*:
   3.1 $A \subseteq B \implies \lambda^*(A) \leq \lambda^*(B)$.
   3.2 $\lambda^*\left(\bigcup_n A_n\right) \leq \sum_n \lambda^*(A_n)$.
   3.3 $\lambda^*(I) = \ell(I)$, for all intervals $I \subseteq \mathbb{R}$.
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