# Last time

1. We gave conditions under which $\lambda^*$ is finitely additive.
2. Introduced the *Carathéodory criterion*: $E$ satisfies CC if for all $W$

$$\lambda^*(W) = \lambda^*(W \cap E) + \lambda^*(W \cap E^c).$$

3. Recall that $\mathcal{M}$ is the collection of sets satisfying CC, we showed that $\mathcal{M}$ is a $\sigma$-algebra that contains the Borel sets.
4. We concluded that $\lambda^*$ restricts to a measure on $\mathcal{M}$. This is the *Lebesgue measure*, and we simply denote it $\lambda$.

# Last time

1. We gave conditions under which $\lambda^*$ is finitely additive.
2. Introduced the *Carathéodory criterion*: $E$ satisfies CC if for all $W$

$$\lambda^*(W) = \lambda^*(W \cap E) + \lambda^*(W \cap E^c).$$

3. Recall that $\mathcal{M}$ is the collection of sets satisfying CC, we showed that $\mathcal{M}$ is a $\sigma$-algebra that contains the Borel sets.
4. We concluded that $\lambda^*$ restricts to a measure on $\mathcal{M}$. This is the *Lebesgue measure*, and we simply denote it $\lambda$.

# Last time

1. We gave conditions under which $\lambda^*$ is finitely additive.
2. Introduced the *Carathéodory criterion*: $E$ satisfies CC if for all $W$

$$\lambda^*(W) = \lambda^*(W \cap E) + \lambda^*(W \cap E^c).$$

3. Recall that $\mathcal{M}$ is the collection of sets satisfying CC, we showed that $\mathcal{M}$ is a $\sigma$-algebra that contains the Borel sets.
4. We concluded that $\lambda^*$ restricts to a measure on $\mathcal{M}$. This is the *Lebesgue measure*, and we simply denote it $\lambda$.

# Last time

1. We gave conditions under which $\lambda^*$ is finitely additive.
2. Introduced the *Carathéodory criterion*: $E$ satisfies CC if for all $W$

$$\lambda^*(W) = \lambda^*(W \cap E) + \lambda^*(W \cap E^c).$$

3. Recall that $\mathcal{M}$ is the collection of sets satisfying CC, we showed that $\mathcal{M}$ is a $\sigma$-algebra that contains the Borel sets.
4. We concluded that $\lambda^*$ restricts to a measure on $\mathcal{M}$. This is the *Lebesgue measure*, and we simply denote it $\lambda$.

# Today

1. We will look at more general measure theory.
2. Prove that all measures have certain nice properties (“continuity”, subadditivity).
3. Introduce real values measurable functions, generalizations of Borel measurable functions, and show they form an algebra.
4. Introduce complex and extended real valued measurable functions.
5. Introduce the notion of $\mu$-almost everywhere and complete measure space.
6. Look at a mistake I made!

# Today

1. We will look at more general measure theory.
2. Prove that all measures have certain nice properties ("continuity", subadditivity).
3. Introduce real values measurable functions, generalizations of Borel measurable functions, and show they form an algebra.
4. Introduce complex and extended real valued measurable functions.
5. Introduce the notion of $\mu$-almost everywhere and complete measure space.
6. Look at a mistake I made!

# Today

1. We will look at more general measure theory.
2. Prove that all measures have certain nice properties ("continuity", subadditivity).
3. Introduce real values measurable functions, generalizations of Borel measurable functions, and show they form an algebra.
4. Introduce complex and extended real valued measurable functions.
5. Introduce the notion of $\mu$-almost everywhere and complete measure space.
6. Look at a mistake I made!

# Today

1. We will look at more general measure theory.
2. Prove that all measures have certain nice properties ("continuity", subadditivity).
3. Introduce real values measurable functions, generalizations of Borel measurable functions, and show they form an algebra.
4. Introduce complex and extended real valued measurable functions.
5. Introduce the notion of $\mu$-almost everywhere and complete measure space.
6. Look at a mistake I made!

# Today

1. We will look at more general measure theory.
2. Prove that all measures have certain nice properties ("continuity", subadditivity).
3. Introduce real values measurable functions, generalizations of Borel measurable functions, and show they form an algebra.
4. Introduce complex and extended real valued measurable functions.
5. Introduce the notion of $\mu$-almost everywhere and complete measure space.
6. Look at a mistake I made!

# Exercise 2d from week 1

## Exercise

Find a strictly positive continuous function $h\colon [0,\pi] \to \mathbb{R}$ such that

$$\int_0^\pi |x - sin(x)| h(x)\, dx \geq 10 \quad \text{and} \quad \int_0^\pi h(x)\, dx = 1.$$

# Exercise 2d from week 1

## Exercise

Find a strictly positive continuous function $h\colon [0,\pi] \to \mathbb{R}$ such that

$$\int_0^\pi |x - sin(x)| h(x)\, dx \geq 10 \quad \text{and} \quad \int_0^\pi h(x)\, dx = 1.$$

# Exercise 2d from week 1

## Exercise

Find a strictly positive continuous function $h\colon [0,\pi] \to \mathbb{R}$ such that

$$\int_0^\pi |x - sin(x)| h(x)\, dx \geq 10 \quad \text{and} \quad \int_0^\pi h(x)\, dx = 1.$$

## "Solution"

- Note that $\sup\{|x - sin(x)| \mid x \in [0,\pi]\} = \pi$.
- $$\int_0^\pi |x - sin(x)| h(x)\, dx \leq \int_0^\pi \pi h(x)\, dx = \pi.$$

# Exercise 2d from week 1

## Exercise

Find a strictly positive continuous function $h\colon [0,\pi] \to \mathbb{R}$ such that

$$\int_0^\pi |x - sin(x)| h(x)\, dx \geq 10 \quad \text{and} \quad \int_0^\pi h(x)\, dx = 1.$$

## "Solution"

- Note that $\sup\{|x - sin(x)| \mid x \in [0,\pi]\} = \pi$.
- $$\int_0^\pi |x - sin(x)| h(x)\, dx \leq \int_0^\pi \pi h(x)\, dx = \pi.$$