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1. We introduced simple functions and the integral of them.
2. Showed that every non-negative extended real valued $\mathcal{A}$-measurable function is a limit of simple functions.
3. Defined the integral of a non-negative extended real valued $\mathcal{A}$-measurable function.
4. Showed basic properties of the integral.
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   (b) If $E \in \mathcal{A}$ and $\mu(E) = 0$ then $\int_E f \, d\mu = 0$.
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1. Two big goals for today
   (a) Show that $\int_E f + g \, d\mu = \int_E f \, d\mu + \int_E g \, d\mu$.
   (b) Monotone Convergence Theorem: If (f_n) is a non-decreasing sequence of non-negative, extended real valued, $\mathcal{A}$-measurable functions, then

$$\lim_n \int_E f_n \, d\mu = \int_E \lim_n f_n \, d\mu.$$

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