

Last time

1. Showed the Dominated Convergence Theorem (DCT): Let $\{f_n\}$ be a sequence of functions that converges pointwise. If there exists a non-negative function g such that

(a) $|f_n| \leq g$ for all n , and

(b) g is integrable,

then

$$\int_E \lim_n f_n d\mu = \lim_n \int_E f_n d\mu,$$

for all $E \in \mathcal{A}$.

2. Talked about important corollaries of the convergence theorems: If $\{E_n\}$ is an increasing sequence of measurable sets with $E = \cup_n E_n$ and f is either non-negative or integrable, then

$$\int_E f d\mu = \lim_n \int_{E_n} f d\mu$$

3. Looked at an example of how to compute with the Lebesgue integral: The function

$$f(x) = \frac{\sin(x)}{x}$$

is not integrable on $(0, \infty)$.

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$$\lim_n \mu(\{x \in \Omega \mid |f_n(x) - f(x)| > \varepsilon\}) = 0$$

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