Last time

1. Defined the integral of real valued functions and complex functions.
2. If $f : \Omega \to \mathbb{R}$ is $\mathcal{A}$-measurable then f is integrable if

$$\int |f| \, d\mu < \infty.$$

   In this case we define the integral of f to be

$$\int f \, d\mu = \int f^+ \, d\mu - \int f^- \, d\mu.$$

3. We denote the set of integrable functions by $\mathcal{L}^1(\Omega, \mathcal{A}, \mu)$.
4. Showed basic properties of the integral: Linearity, monotonicity, triangle inequality ...
5. Showed that the Lebesgue integral is an extension of the Riemann integral for continuous functions on closed bounded intervals.

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$$\int_E \liminf_n f_n \, d\mu \leq \liminf_n \int_E f_n \, d\mu.$$

2. Show the Dominated Convergence Theorem (DCT): Suppose that $\{f_n\}$ is a sequence of complex valued $\mathcal{A}$-measurable functions that converges pointwise. If there exists an non-negative, extended real valued, $\mathcal{A}$-measurable function g such that
   (a) $|f_n| \leq g$ for all n, and
   (b) g is integrable,

   then

$$\int_E \lim_n f_n \, d\mu = \lim_n \int_E f_n \, d\mu,$$

   for all $E \in \mathcal{A}$.

3. Look at examples of how to compute with the Lebesgue integral.

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