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2. Defined L^p to be the quotient space of $\mathcal{L}^p$ by the equivalence relation defined by $f \sim g$ if $f = g$ μ-a.e.
3. We showed that L^p is a normed space under the norm

$$\|[f]\|_p = \left(\int |f|^p \, d\mu \right)^{\frac{1}{p}}.$$

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Today and moving forward

1. We wish to integrate functions $f\colon \mathbb{R}^2 \to \mathbb{C}$, or more generally functions $f\colon \Gamma \times \Lambda \to \mathbb{C}$ where $(\Gamma, \mathcal{S}, \mu)$ and $(\Lambda, \mathcal{T}, \nu)$ are measure spaces.
2. We want a σ-algebra $\mathcal{S} \otimes \mathcal{T}$ on $\Gamma \times \Lambda$ that contains sets of the form $S \times T$, $S \in \mathcal{S}, T \in \mathcal{T}$.
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