

Last time

1. We showed that \mathcal{L}^p is a linear space for $1 \leq p \leq \infty$.
2. We proved two classical inequalities.

Hölder: If $1 \leq p, q \leq \infty$ are such that $\frac{1}{p} + \frac{1}{q} = 1$ then

$$\int |fg| d\mu \leq \|f\|_p \|g\|_q.$$

Minkowski: If $1 \leq p \leq \infty$ and $f, g \in \mathcal{L}^p$ then

$$\|f + g\|_p \leq \|f\|_p + \|g\|_p.$$

3. We discussed the quotient space L^p , which is \mathcal{L}^p but with functions that are equal μ -a.e. identified.
4. We showed that the p -norm is a norm on L^p .

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1. We will show that L^p is complete in $\|\cdot\|_p$ for $1 \leq p \leq \infty$.
2. We will show that L^2 is a Hilbert space with inner product

$$\langle f, g \rangle = \int f \bar{g} d\mu.$$

3. We will look at two important examples.
 - 3.1 L^2 of the measure space $(\mathbb{N}, \mathcal{P}(\mathbb{N}), \mu)$ where μ is the counting measure.
 - 3.2 L^2 of the measure space $([0, 2\pi], \mathcal{B}, \lambda)$

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