

Errata to Tom L. Lindstrøm: *Spaces. An Introduction to Real Analysis*

This list contains all reported misprints and errors (many thanks to Eirik Ergon Aung, Ole Fredrik Brevig, Ulrik Skre Fjordholm, Simon Foldvik, Camilla Green, Marius Havgar, Tobias Laundal, Bernt Ivar Nødland, Marc Paoella, Gaute Schwartz, Suman Sunderroy, Håvard Syvertsen, Erlend Fornæss Wold, and (especially) Olav Skutlaberg). The ones that are most likely to cause confusion are listed in **red**, and comments on the corrections are in **blue** to distinguish them from the corrections themselves.

There is slight inconsistency in the notation for open balls: According to the definition on page 49, they are to be denoted by $B(a; r)$, but in parts of the book this degenerates into $B(a, r)$. I have not attempted to trace these inconsistencies.

Negative line numbers are counted from below, hence “page 41, line -9” means line number 9 from the bottom of page 41.

| Where | Says | Should have said |
|---|--|--|
| page 12, line -6 | the | then |
| page 15, line 16 | $\subseteq f(\bigcup_{A \in \mathcal{A}} A)$ | $\subseteq f(\bigcup_{A \in \mathcal{A}} A)$ |
| page 16, line 6 | $f^{-1}(B^c)$ | $f^{-1}(B^c)$ |
| page 17, line 9 | $B \subseteq Y$ | $B \subseteq f(X)$ |
| page 20, line 5 | $z \sim y$ | $z \sim w$ |
| page 22, line 11 | $A_1 \times A_2 \times \dots \times A_n$ | $A_1 \times A_2 \times \dots \times A_n$ |
| page 25, line 14 | $x_n = a..$ | $x_n = a.$ |
| page 29, line 13 | $\min\{\delta_1, \delta_2.\}$ | $\min\{\delta_1, \delta_2\}$ |
| page 29, line -6 | that if $f(x) = \sqrt{x}$ | that $f(x) = \sqrt{x}$ |
| page 29, line -5 | $ \ \mathbf{a}\ - \ \mathbf{b}\ \leq \ \mathbf{a} - \mathbf{b}\ $ | $ \ \mathbf{a}\ - \ \mathbf{b}\ \leq \ \mathbf{a} - \mathbf{b}\ $ |
| page 40, line -15 | maximum | minimum |
| page 43, line 9 | leads | lead |
| page 45, line 9 | $d(x, y) = y_1 - x_1 + y_2 - x_1 $ | $d(x, y) = y_1 - x_1 + y_2 - x_2 $ |
| page 65, line 6 | K | K |
| page 70, lines 11 | without out | without |
| page 70, line -14 | $O \in \mathcal{B}$ | $O \in \mathcal{O}$ |
| page 71, Problem 7 | $\neq \emptyset$ | $= \emptyset$ |
| page 72, lines 10-11 | ... is a metric space $(\bar{X}, d_{\bar{X}})$ | ... is a complete metric space $(\bar{X}, d_{\bar{X}})$ |
| page 72, line 14 | dense | dense in |
| page 82, lines -2 | <i>if</i> | if |
| page 84, lines -6 | maxium (twice) | maximum |
| page 86, Figure 4.3.1 label on y -axis | n | n |
| page 87, line 16 | such that $ f(t) - f_n(t) $ | such that if $n \geq N$, then $ f(t) - f_n(t) $ |
| page 87, line -13 | , <i>leq</i> | $ \leq$ |
| page 88, lines 8 and 10 | partial sum | partial sums |
| page 90, line 13 | se | see |
| page 92, line 9 | $\frac{\ln x}{n^x}$ | $\frac{\ln n}{n^x}$ |
| page 99, line -8 | $C(X, Y)$ | $B(X, Y)$ |
| page 100, line 18 | Y | Y |
| page 105, line 13 | $\mathbf{y}_0 + \int_0^t \mathbf{f}(t, \mathbf{z}(t)) dt$ | $\mathbf{y}_0 + \int_0^t \mathbf{f}(s, \mathbf{z}(s)) ds$ |

| Where | Says | Should have said |
|--|---|---|
| page 110, line -6 | ... get at function ... | ... get a function ... |
| page 115, line -19 | $\ \int_0^t \dots \ $ | $\ \int_0^t \dots ds \ $ |
| page 118, line -9 (to ensure strict inequality in line -2) | $E(\dots) \leq E(\dots) \leq \frac{\epsilon}{2}$ | $E(\dots) < E(\dots) \leq \frac{\epsilon}{2}$ |
| page 120, line 12 | $\dots > \frac{c_n}{\sqrt{n}}$ | $\dots > \frac{c_n}{\sqrt{n}}$ |
| page 127, line 7 (spacing) | $\dots g_x(y) < f(y) + \epsilon$ | $\dots g_x(y) < f(y) + \epsilon$ |
| page 138, line -7 and -9 | x | \mathbf{x} |
| page 139, line -6 | $x_1 \mathbf{e}_1 + x_2 \mathbf{e}_2 + \dots x_n \mathbf{e}_n$ | $x_1 \mathbf{e}_1 + x_2 \mathbf{e}_2 + \dots + x_n \mathbf{e}_n$ |
| page 141, line -7 | Example 3 | Example 1 |
| page 144, Figure 5.3.3 | $\mathbf{u} - \mathbf{p}$ | the drawing lacks an arrow (vector) |
| page 150, line 15 | $\sum_{n=1}^N x_n y_n = \dots$ | $\sum_{n=1}^N x_n y_n = \dots$ |
| page 152, line -13 | ... exist a \mathbf{u}_n such ... | ... exist a \mathbf{u}_n such ... |
| page 153, line -7 | $\leq \sup \left\{ \frac{\ A(\mathbf{u})\ + \dots}{\ \mathbf{u}\ _V} \right.$ | $\leq \sup \left\{ \frac{\ A(\mathbf{u})\ _W + \dots}{\ \mathbf{u}\ _V} \right.$ |
| page 155, line 14 | $\ker A$ | $\ker(A)$ |
| page 155, line -21 | Theorem | Theorem |
| page 157, line -13 | $\ f\ = 1$ | $\ f\ \leq 1$ (to include $n = 1$) |
| page 160, line 13 | $g: [a, b] \rightarrow [a, b]$ | $g: [a, b] \rightarrow \mathbb{R}$ |
| page 160, line -4 | there is a $c \in \mathbb{R}$ | there is a positive $c \in \mathbb{R}$ |
| page 163, line 11 | $\overline{B}(0, \mathbf{n})$ | $\overline{B}(0, \mathbf{n})$ |
| page 171, line -6 | due to due to Banach | due to Banach |
| page 180, line 5 When the spaces X, Y are complex, we have to let t go to 0 as a complex variable. | | |
| page 180, line 8 | X is a normed space | X, Y are normed spaces |
| page 180, line 12 | $tr \in O$ | $\mathbf{a} + tr \in O$ |
| page 181, line 2 | $\mathbf{F}(\mathbf{a})'(1)$ | $\mathbf{F}'(\mathbf{a})(1)$ |
| page 182, line 17 | exists | exist |
| page 190, line 14 | $\leq \frac{\epsilon}{3} \mathbf{r}$ | $\leq \frac{\epsilon}{3} \ \mathbf{r}\ $ |
| page 191, line -8 | normed space | complete normed space |
| page 191, lines 8, -12, -7, -1 | $ R(\dots) - R(\dots) $ | $\ R(\dots) - R(\dots)\ $ |
| page 192, line 1 (twice) | $ R(\dots) - R(\dots) $ | $\ R(\dots) - R(\dots)\ $ |
| page 195, line 7 | $\mathbf{X} = \mathbb{R}^d$ | $X = \mathbb{R}^d$ |
| page 195, line 19 | $F^{(k)}$ | $\mathbf{F}^{(k)}$ |
| page 196, line -3 | $H(t) = \int_0^t \frac{1}{n!} (1-t)^n \mathbf{F}^{(n+1)}(t) dt$ | $H(t) = \int_0^t \frac{1}{n!} (1-s)^n \mathbf{F}^{(n+1)}(s) ds$ |
| page 199, Example 1 Here I must have forgotten to divide the terms by $ \alpha !$. This doesn't change the first order terms, but the second order terms have to be divided by $2! = 2$ and the third order terms by $3! = 6$. | $f(\mathbf{a}) + \frac{\partial f}{\partial x_1}(\mathbf{a})h_1 + \frac{\partial f}{\partial x_2}(\mathbf{a})h_2$ $+ \frac{\partial^2 f}{\partial x_1^2}(\mathbf{a})h_1^2 + 2 \frac{\partial^2 f}{\partial x_1 \partial x_2}(\mathbf{a})h_1 h_2$ $+ \frac{\partial^2 f}{\partial x_2^2}(\mathbf{a})h_2^2 + \frac{\partial f^3}{\partial x_1^3}(\mathbf{a})h_1^3$ $+ 3 \frac{\partial f^3}{\partial x_1^2 \partial x_2}(\mathbf{a})h_1^2 h_2 +$ $+ 3 \frac{\partial f^3}{\partial x_1 \partial x_2^2}(\mathbf{a})h_1 h_2^2 + \frac{\partial^3 f}{\partial x_2^3}(\mathbf{a})h_2^3$ | $f(\mathbf{a}) + \frac{\partial f}{\partial x_1}(\mathbf{a})h_1 + \frac{\partial f}{\partial x_2}(\mathbf{a})h_2$ $+ \frac{1}{2} \frac{\partial^2 f}{\partial x_1^2}(\mathbf{a})h_1^2 + \frac{\partial^2 f}{\partial x_1 \partial x_2}(\mathbf{a})h_1 h_2$ $+ \frac{1}{2} \frac{\partial^2 f}{\partial x_2^2}(\mathbf{a})h_2^2 + \frac{1}{6} \frac{\partial f^3}{\partial x_1^3}(\mathbf{a})h_1^3$ $+ \frac{1}{2} \frac{\partial f^3}{\partial x_1^2 \partial x_2}(\mathbf{a})h_1^2 h_2 +$ $+ \frac{1}{2} \frac{\partial f^3}{\partial x_1 \partial x_2^2}(\mathbf{a})h_1 h_2^2 + \frac{1}{6} \frac{\partial^3 f}{\partial x_2^3}(\mathbf{a})h_2^3$ |
| page 200, line 1 | ... with $ \alpha = k + 1$ | ... with $ \alpha = n + 1$ |
| page 200, line 2 | $ f(\mathbf{a} + \mathbf{a}) - \dots $ | $ f(\mathbf{a} + \mathbf{h}) - \dots $ |
| page 205, line 9 | $\frac{\partial \mathbf{F}_i}{\partial x_j}$ | $\frac{\partial \mathbf{F}_i}{\partial x_j}$ |
| page 205, lines -9, -10 | x | \mathbf{x} |

| Where | Says | Should have said |
|---|--|--|
| page 206, line 14 | $g'(y_0) = \frac{1}{f'(x_0)}$ | $g'(y_0) = \frac{1}{f'(x_0)}$ |
| page 207, line -5 | $\mathbf{x}_1 - \mathbf{x}_2 = \mathbf{H}(\mathbf{x}_1) - \mathbf{H}(\mathbf{x}_2)$ | $\mathbf{x}_1 - \mathbf{x}_2 = \mathbf{H}(\mathbf{x}_2) - \mathbf{H}(\mathbf{x}_1)$ |
| page 209-210, proof of the Inverse Function Theorem | | At the end of the proof one should check that \mathbf{G} really is local inverse of \mathbf{F} . This can be left to the reader, but the need for a proof should perhaps be pointed out. |
| page 210, line 20 | A is $\mathbf{F}'(\bar{\mathbf{a}})$ | A is $\mathbf{F}'(\mathbf{a})$ |
| page 212, line -13 | $\mathbf{G}(x)$ | $\mathbf{G}(\mathbf{x})$ (two places) |
| page 212, line -7 | $\mathbf{H}'(\mathbf{a})$ | $\mathbf{H}'(\mathbf{a}, \mathbf{b})$ |
| page 213, line -15 | $\mathbf{X} \times \mathbf{Y}$ | $X \times Y$ |
| page 215, line 7 | Corollary 6.8.3 | Corollary 6.8.2. |
| page 224, line -13 | \mathbf{y}_x | \mathbf{y}_x |
| page 225, line 2 | $= \mathbf{y}_2(t)$ | $= \mathbf{y}_2(t_0)$ |
| page 226, line 10 | $A: X_1 \times X_2 \times \dots \times X_n \rightarrow Y$ | $A: X_1 \times X_2 \times \dots \times X_n \rightarrow Y$ |
| page 226, line -2 | X_1, X_2, \dots, X_n | X_1, X_2, \dots, X_n |
| page 227, lines 7, 13, 14, -10 | $X_1 \times X_2 \times \dots \times X_n$ | $X_1 \times X_2 \times \dots \times X_n$ |
| page 228, line 9 | $\dots, \ \mathbf{x} - \mathbf{a}_2\ $ | $\dots, \ \mathbf{x} - \mathbf{a}_n\ $ |
| page 229, lines -10 and -5 | $A(\mathbf{F}_1(\mathbf{a}), \mathbf{F}_2(\mathbf{a}), \dots, \dots, \mathbf{F}'_{n-1}(\mathbf{a})(\mathbf{r}), \mathbf{F}_{n-1}(\mathbf{a}))$ | $A(\mathbf{F}_1(\mathbf{a}), \mathbf{F}_2(\mathbf{a}), \dots, \dots, \mathbf{F}'_{n-1}(\mathbf{a})(\mathbf{r}), \mathbf{F}_n(\mathbf{a}))$ |
| page 229, line -8 | $\mathbf{K}(\mathbf{x}) = (\mathbf{F}_1(\mathbf{x}), \mathbf{F}_2(\mathbf{x}), \dots, \mathbf{F}_n(\mathbf{x}))$ | $\mathbf{K}(\mathbf{x}) = (\mathbf{F}_1(\mathbf{x}), \mathbf{F}_2(\mathbf{x}), \dots, \mathbf{F}_n(\mathbf{x}))$ |
| page 231, line 17 | derivates | derivatives |
| page 231, line 18 | derivate | derivative |
| page 232, line 3 | \mathbf{X}^n | X^n |
| page 232, line 18 | $\mathcal{L}(X \rightarrow \mathcal{L}(X, \dots, \mathcal{L}(X, Y) \dots))$ | $\mathcal{L}(X, \mathcal{L}(X, \dots, \mathcal{L}(X, Y) \dots))$ |
| page 233-235, proof of Theorem 6.11.3. The order of (\mathbf{r}) and (\mathbf{s}) has become mixed up several times here. The problem is that I say (p. 233, l. -8) that I shall prove something for $\mathbf{F}''(\mathbf{a})(\mathbf{r})(\mathbf{s})$ and then prove it for $\mathbf{F}''(\mathbf{a})(\mathbf{s})(\mathbf{r})$ instead. The next three corrections sort things out. | | |
| page 233, line -8 | $\mathbf{F}''(\mathbf{a})(\mathbf{r})(\mathbf{s})$ | $\mathbf{F}''(\mathbf{a})(\mathbf{s})(\mathbf{r})$ |
| page 233, line -7 | $\mathbf{F}''(\mathbf{a})(\mathbf{s})(\mathbf{r})$ | $\mathbf{F}''(\mathbf{a})(\mathbf{r})(\mathbf{s})$ |
| page 235, line 7 | $\mathbf{F}''(\mathbf{a})(\mathbf{r})(\mathbf{s})$ | $\mathbf{F}''(\mathbf{a})(\mathbf{s})(\mathbf{r})$ |
| page 242, line 1 | $A_1, A_2, A_3 \dots$ | A_1, A_2, A_3, \dots |
| page 242, line 3 | $\mu(\bigcup_{n=1}^{\infty} A_N) =$ | $\mu(\bigcup_{n=1}^{\infty} A_n) =$ |
| page 245, line -5 | for $n > 1$ | for $n > 1$ |
| page 245, line -4 | for all N | for all n |
| page 246, line 9 | subsets of A | subsets of \mathbb{N} |
| page 250, line -19 | This an instance | This is an instance |
| page 253, line -11 | $(f^{-1}([-\infty, s]))^c$ | $(f^{-1}([-\infty, s]))^c$ |
| page 253, line -9 | $(f^{-1}([-\infty, s]))^c$ | $(f^{-1}([-\infty, s]))^c$ |
| page 255, line 4 | $\{x \in X \mid (f + g) < r\}$ | $\{x \in X \mid (f + g)(x) < r\}$ |
| page 255, line 5 | $\{x \in X \mid g < r - q\}$ | $\{x \in X \mid g(x) < r - q\}$ |

| Where | Says | Should have said |
|---------------------|--|---|
| page 257, line 8 | almost everywhere | on a null set Comment: The text only defines equality a.e. for measurable functions which doesn't make sense in part b). |
| page 260, line -11 | step function | simple function |
| page 261, line 2 | $\int f d u =$ | $\int f d \mu =$ |
| page 261, line -11 | $\sum_{i=1}^n$ | $\sum_{i=1}^n$ |
| page 261, line -10 | $\int_{A_n} a d u = a m$ | $\int_{A_n} a d u \geq a m$ |
| page 261, line -2 | $g(x) = \sum_{i=1}^m b_i \mathbf{1}_{B_i}$ | $g(x) = \sum_{i=1}^m b_i \mathbf{1}_{B_i}$ |
| page 266, line -11 | $g_k(x) = \inf_{n \geq k} f_n(x)$ | $g_k(x) = \inf_{n \geq k} f_n(x)$ |
| page 268, line -7 | $\lim_{n \rightarrow \infty} \phi_{\mathcal{P}_n} = \lim_{n \rightarrow \infty} \Phi_{\mathcal{P}_n}$ a.e. | $\lim_{n \rightarrow \infty} \phi_{\mathcal{P}_n} = \lim_{n \rightarrow \infty} \Phi_{\mathcal{P}_n}$ a.e. (see e.g. Exercise 4b) |
| page 268, line -4 | a.s. | a.e. (Comment: a.s. is short for <i>almost surely</i> and is an alternative expression for a.e. that hasn't been introduced in the text). |
| page 269, line 16 | measurable functions | integrable functions |
| page 271, line 4 | nonnegative functions | nonnegative, measurable functions |
| page 274, line -2 | for each $x \in X$ | for each $x \in \mathbb{R}$ |
| page 278, line 8 | with inequality | with equality |
| page 279, line 2 | $\frac{ f(x) ^p}{\ f\ _p^p}$ | $\frac{ f(x) ^p}{\ f\ _p^p}$ |
| page 281, line -10 | $= \lim_{N \rightarrow \infty} \ \sum_{n=1}^N \mathbf{u}_n\ _p \leq \dots$ | $= \lim_{N \rightarrow \infty} \ \sum_{n=1}^N \mathbf{u}_n \ _p \leq \dots$ |
| page 283, line 11 | $p, q \in [0, \infty]$ | $p, q \in [1, \infty]$ |
| page 287, line 2 | pointwise to f on A_K | pointwise to f on A_K^c |
| page 287, line 8 | $p \in [0, \infty)$ | $p \in [1, \infty)$ |
| page 287, line 14 | $\mu(A) < \infty$ | $\mu(X) < \infty$ |
| page 292, line -8 | subsets of \mathbb{R}^d | subsets of X |
| page 293, Problem 4 | Let $(X, \mathcal{R}, \bar{\mu})$ be the completion of (X, \mathcal{R}, μ) . Show that $\mu^*(A) = \bar{\mu}(A)$ | Let $(X, \mathcal{R}, \bar{\rho})$ be the completion of (X, \mathcal{R}, ρ) . Show that $\mu^*(A) = \bar{\rho}(A)$ |
| page 295, line 12 | $A \subseteq \mathbb{R}^d$ | $A \subseteq X$ |
| page 296, line 13 | $A \in \mathcal{A}$ | $A \subseteq X$ |
| page 297, line -3 | measure extension of \mathcal{R} | measure extension of ρ |
| page 299, line 11 | | Added explanation: In the first inequality we are using that a premeasure ρ on an algebra is increasing; i.e. $\rho(A) \leq \rho(B)$ when $A \subseteq B$. |
| page 299, line 13 | If μ is σ -finite | If (X, \mathcal{M}, μ) is σ -finite |
| page 299, line -7 | $C_n \setminus (C_1 \cup \dots \cup C_{n-1})$ | $C_n \setminus (C_1 \cup \dots \cup C_{n-1})$ |
| page 300, line 9 | R^c is a disjoint union | R^c is a finite, disjoint union |
| page 301, line -13 | $\sum_{i=1}^n \lambda(R_i) = \dots$ | $\sum_{i=1}^{\infty} \lambda(R_i) = \dots$ |
| page 302, line 5 | $A = \cup_{j=1}^M R_j$ | $A = \cup_{j=1}^M R_j$ |
| page 310, line 10 | g is continuous | g is continuous |
| page 310, line -5 | $p \in [0, \infty)$ | $p \in [1, \infty)$ |
| page 329, line -13 | $\dots - \frac{1}{2\pi} \left[\frac{e^{-inx}}{n^2} \right]_{-\pi}^{\pi} = \dots$ | $\dots + \frac{1}{2\pi} \left[\frac{e^{-inx}}{n^2} \right]_{-\pi}^{\pi} = \dots$ |
| page 334, line -16 | $(C_p, \ \cdot\)$ | $(C_P, \ \cdot\)$ |
| page 340, line-2 | $D_n(t) = \frac{\sin((n+\frac{1}{2})t)}{\sin \frac{t}{2}}$ To ... | $D_n(t) = \frac{\sin((n+\frac{1}{2})t)}{\sin \frac{t}{2}}$. To ... |
| page 346, line 13 | $<$ | \leq (first inequality) |
| page 346, line 18 | C_p | C_P |
| page 357, line -13 | C_p | C_P |