MAT3400/4400 - Spring 2023 - Exercises for Friday, Mars 31

- From the exercises in Chap. 1 of the notes on ELA: 4, 5, 6

- From the exercises in Chap. 2 of the notes on ELA: 3, 4, 6, 7, 9

Extra exercise 26

Let $p \in [1, \infty)$ and $f : \mathbb{R} \to \mathbb{R}$ be the continuous function given by

$$f(x) = \frac{2 + \sin(x)}{\sqrt{x^2 + 1}} \quad \text{for all } x \in \mathbb{R}.$$

Let $\mathcal{A}$ denote the σ-algebra of all Borel subsets of $\mathbb{R}$ and μ denote the Lebesgue measure on $(\mathbb{R}, \mathcal{A})$.

Show that $f \in \mathcal{L}^p(\mathbb{R}, \mathcal{A}, \mu)$ if and only if $p > 1$.