MAT3400/4400 - Spring 2023 - Exercises for Friday, Feb. 3

- From Lindstrøm's book, section 7.1 : 10, 15, 16

- From Lindstrøm's book, section 7.2 : 1, 4, 5

Extra-exercise 4

Let $\mathcal{J}$ be the collection of subsets of $\mathbb{R}$ given by

$\mathcal{J} = \{\emptyset\} \cup \{(a,b] : a,b \in \mathbb{R}, a < b\} \cup \{(-\infty, b] : b \in \mathbb{R}\} \cup \{(a,\infty) : a \in \mathbb{R}\}.$

Recall that $\mathcal{B}_{\mathbb{R}}$ denotes the Borel σ-algebra on $\mathbb{R}$. Show that

$$\mathcal{B}_{\mathbb{R}} = \sigma(\mathcal{J}),$$

i.e., $\mathcal{B}_{\mathbb{R}}$ is the least σ-algebra on $\mathbb{R}$ containing $\mathcal{J}$.

- From Lindstrøm's book, section 7.3 : 3 (as a sample, show one of the implications.)

Extra-exercise 5

Let $(X, \mathcal{A})$ be a measurable space and let $f : X \to \overline{\mathbb{R}}$.

a) Assume that f is constant on X. Check that f is measurable (w.r.t. $\mathcal{A}$).

b) Assume that $\mathcal{A} = \{\emptyset, X\}$ (and note that it is almost immediate that $\mathcal{A}$ is a σ-algebra on X).

Show that if f is measurable (w.r.t. $\mathcal{A}$), then f is constant on X.

Extra-exercise 6

Let $(X, \mathcal{A})$ be a measurable space and let $f : X \to \mathbb{R}$. Show that f is measurable (w.r.t. $\mathcal{A}$) if and only if $f^{-1}(B) \in \mathcal{A}$ for every $B \in \mathcal{B}_{\mathbb{R}}$.

(*Hint.* For the implication ($\Rightarrow$), use Exercise 7.1.10).