MAT3400/4400 - Spring 2023 - Exercises for Friday, Feb. 24

• From Lindstrøm's book, section 7.5 : 15 (NB: all the f_n's in this exercise are supposed to be nonnegative)

• From Lindstrøm's book, section 7.6 : 1, 3, 5, 6, 7, 9

Extra exercise 12

Let $(X, \mathcal{A}, \mu)$ be a measure space and $\rho : X \to [0, \infty]$ be measurable. We recall that $\nu : \mathcal{A} \to [0, \infty]$ given by $\nu(A) = \int_A \rho \, d\mu$ for each $A \in \mathcal{A}$ is then a measure on $(X, \mathcal{A})$.

Let $f : X \to \overline{\mathbb{R}}$ be measurable. Use Extra Exercise 10 to show that f is integrable (w.r.t. ν) if and only if $f\rho$ is integrable (w.r.t. μ), in which case we have

$$\int_A f \, d\nu = \int_A f \, \rho \, d\mu$$

for all $A \in \mathcal{A}$.