

## MAT3400/4400 - Spring 2023 - Exercises for Friday, Mars 3

NB: Since you should also work with the mandatory assignment, we give less exercises than usual this week.

### Extra exercise 13

Consider the Lebesgue measure  $\mu$  on the Borel  $\sigma$ -algebra on  $[1, \infty)$ , and let  $f : [1, \infty) \rightarrow \mathbb{R}$  be the measurable function given by

$$f = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \mathbf{1}_{[n, n+1)} \quad (\text{pointwise}).$$

Show that  $\lim_{m \rightarrow \infty} \int_{[1, m]} f d\mu$  exists. Is  $f$  integrable w.r.t.  $\mu$ ?

- From Lindstrøm's book, section 7.6: 8
- From Lindstrøm's book, section 8.1: 1, 2, 4