Exercises for April 18 - 24

14.3, 14.10, 14.14, 14.18(i)-(iii).

Additional exercise.

(1) Assume $\phi\colon [0,+\infty) \to [0,+\infty)$ is a convex function. Define $\phi(+\infty) = +\infty$. Show that $\phi\colon [0,+\infty] \to [0,+\infty]$ is convex in the sense that $\phi(\lambda x + (1-\lambda)y) \le \lambda\phi(x) + (1-\lambda)\phi(y)$ for all $x, y \in [0,+\infty]$ and $0 < \lambda < 1$.

(2) Consider a probability measure space $(X, \mathcal{B}, \mu)$ and the constant convex function $\phi = 1$ on $[0,+\infty)$. Extend ϕ to $[0,+\infty]$ as above by $\phi(+\infty) = +\infty$. Assume $f\colon X \to [0,+\infty)$ is a nonintegrable measurable function on X. Then

$$\phi\Big(\int_X f\,d\mu\Big) = +\infty \quad \text{and} \quad \int_X \phi\circ f\,d\mu = 1,$$

contradicting Theorem 13.13 in the book. Where is a mistake in the proof?