

## TEACHING PLAN

### MAT4410 - Advanced Linear Analysis, Autumn 2025

Lecturer: Nadia Larsen

Literature for the course: The book "An introduction to functional analysis" by James C. Robinson, hereafter referred to as [R]. Supplementary notes by Erik Bédos, hereafter referred to as [B].

#### Week 34

**Lecture 1, 19.08** Presentation of the course. Finite-dimensional spaces, [R, chp 5].

**Lecture 2, 20.08** Extension by continuity. Hilbert spaces, [B, Chp 1.3], [R, 9.1-9.3].

#### Week 35

**Lecture 3, 26.08** Hilbert spaces: [R, chp 9.3-9.5, 10.1-10.3], [B, chp 2.2]

**Lecture 4, 27.08** Hilbert spaces: [B, chp 2.3], [R, chp 10.1-10.3]. Exercises: 5.1, 5.4 in [R], 1.2, 1.6 in [B].

#### Week 36

**Lecture 5, 2.09** Adjoint operators: [B, 2.4-2.5], [R, 13.1-13.2].

**Lecture 6, 3.09** Adjoint operators: [B, 2.4-2.5], [R, 13.1-13.2]. Exercises: 10.9, 10.10 in [R], 2.6, 2.7 in [B].

#### Week 37

**Lecture 7, 9.09** Self-adjoint operators, unitary operators. [B, 2.5-2.6].

**Lecture 8, 10.09** Exercises from previous weeks. In addition, exercises 13.1, 13.2, 13.5. from [R] and 2.20 from [B].

#### Week 38

**Lecture 9, 16.09** Compact operators: [R, 5.1-5.3], [B, 4.1].

**Lecture 10, 17.09** Compact operators: [R, 5.1-5.3], [B, 4.2].

#### Week 39

**Lecture 11, 23.09** The spectral theorem for compact self-adjoint operators. [R, chp16 ], [B, chp. 4.3].

**Lecture 12, 24.09** The spectral theorem for compact, self-adjoint operators. Exercises: [B, 4.1 b), c), d), 4.2, 4.3, 4.5]. Compactness of the integral operator  $T_K$ , see [B, example 4.1.5], [R, Prop. 15.4].

#### Week 40

**Lecture 13, 30.09** The spectral theorem for compact, self-adjoint operators, [R, chp 16], [B, 4.3.]

**Lecture 14, 01.10** Sturm-Liouville theory, [R, chp 17], [B, chp 5].

**Week 41**

**Lecture 15, 07.10** Sturm-Liouville problems, [R, chp. 17], [B, chp 5].

**Lecture 16, 08.10** We look at examples of the Sturm-Liouville problem, see [B, example 5.4.4]. Exercises: [R, 15.3, 15.9 (see example 11.7), 16.6, 16.7(i)]. Also [B, 4.17, 4.18].

**Week 42**

**Lecture 17, 14.10** Duals of Banach spaces, the Hahn-Banach theorem (for complex spaces) and some applications, [R, chp. 19, 20.1-20.3].

**Lecture 18, 15.10** Quotients of Banach spaces. The Baire category theorem. From [B, 3.2], [R, 22.1].

**Week 43**

**Lecture 19, 21.10** The Baire category theorem and the principle of uniform boundedness, [R, chp 22]. Application to the existence of a continuous periodic function whose Fourier series at 0 fails to converge.

**Lecture 20, 22.10** Three fundamental theorems of functional analysis, [R, chp 23].

**Week 44**

**Lecture 21, 28.10** Applications of the fundamental theorems of functional analysis. Adjoints of linear maps between Banach spaces, the second dual, reflexive spaces [R, 20.4, 26].

**Lecture 22, 29.10** Applications of the fundamental theorems of functional analysis: duals of quotient spaces [B, section 3.6]. The mandatory assignment must be submitted on 30 October.

**Week 45**

**Lecture 23, 4.11** Reflexive spaces, [R, chp 26], [B, section 3.6].

**Lecture 24, 5.11** Exercises from past exams.

**Week 46**

**Lecture 25, 11.11** The Fredholm alternative, [R, chp 24], [B, section 4.4].

**Lecture 26, 12.11** Exercises from past exams.

**Week 47**

**Lecture 27, 18.11** Exercises from past exams.

**Lecture on 19.11** There will be no lecture this day, but I plan to have a lecture on either Thursday, 20 November at 14:15-16:00 or Friday, 21 November at 10:15-12:00. We go over the problems from the assignment and some exam exercises.

**Week 48**

**Lecture on 25.11** Lecture is cancelled.

**Lecture on 26.11** Lecture is cancelled.

**Week 51: Exam on Monday 15.12 at 9:00-13:00.**