8th April, 2026

COURSE MAT4430

Mandatory assignment 1 of 1

Submission deadline

Thursday 23th APRIL 2026, 14:30 in Canvas (canvas.uio.no).

Instructions

Note that you have **one attempt** to pass the assignment. This means that there are no second attempts.

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with LaTeX). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

In exercises where you are asked to write a computer program, you need to hand in the code along with the rest of the assignment. It is important that the submitted program contains a trial run, so that it is easy to see the result of the code.

Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) no later than the same day as the deadline.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

To pass the assignment you are expected to solve at least one of the three problems in this set. You can freely choose what problem to solve, but remember to justify your solution.

Problem 1. (The amplitude damping channel). For a real number $\theta \in [0,1]$ we define a map $N_\theta : B(\mathbb{C}^2) \to B(\mathbb{C}^2)$ by

$$N_\theta \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} = \begin{bmatrix} x_{11} + \theta x_{22} & \sqrt{1-\theta}\, x_{12} \\ \sqrt{1-\theta}\, x_{21} & (1-\theta) x_{22} \end{bmatrix}.$$

(i) Show that N_θ is a linear map. Compute the Choi matrix C_θ of N_θ. Show that N_θ is a quantum channel.

(ii) Compute a Choi-Kraus representation of N_θ.

(iii) Compute a Stinespring dilation of N_θ.

For a complex Euclidean space $\mathcal{H}$, let $1_\mathcal{H}$ be the identity operator in $B(\mathcal{H})$ and let $\mathrm{Id}_\mathcal{H}$ denote the identity map on $B(\mathcal{H})$.

Problem 2. Let $\mathcal{H}$ be a complex Euclidean space. Consider the partial trace map $\mathrm{Id}_\mathcal{H} \otimes \mathrm{Tr} : B(\mathcal{H} \otimes \mathcal{H}) \to B(\mathcal{H})$, given by

$$(\mathrm{Id}_\mathcal{H} \otimes \mathrm{Tr})(X \otimes Y) = \mathrm{Tr}[Y]X,$$

for $X, Y \in B(\mathcal{H})$. Let $|\Omega_\mathcal{H}\rangle$ be the maximally entangled vector in $\mathcal{H} \otimes \mathcal{H}$ and define $|v_Z\rangle := (1_\mathcal{H} \otimes Z)|\Omega_\mathcal{H}\rangle$ for each $Z \in B(\mathcal{H})$.

(i) Show that

$$(\mathrm{Id}_\mathcal{H} \otimes \mathrm{Tr})[|v_Z\rangle\langle v_Z|] = Z^T \overline{Z}$$

for every $Z \in B(\mathcal{H})$, where Z^T is the transpose of Z and $\overline{Z}$ is obtained by taking the complex conjugate in each entry of the matrix of Z with respect to the computational basis.

(ii) Suppose that $\rho \in D(\mathcal{H})$ is a quantum state of $\mathcal{H}$. Show that there exists $Z \in B(\mathcal{H})$ satisfying

$$(\mathrm{Id}_\mathcal{H} \otimes \mathrm{Tr})[|v_Z\rangle\langle v_Z|] = \rho$$

and $\langle Z, Z\rangle_{HS} = 1$, where the inner product is taken with respect to the Hilbert-Schmidt norm.

Problem 3. Let $\mathcal{H} = \mathbb{C}^d$ and define $\Delta : B(\mathbb{C}^d) \to B(\mathbb{C}^d)$ by

$$\Delta(X) = \mathrm{Tr}[X]\frac{1_\mathcal{H}}{d}$$

for every $X \in B(\mathcal{H})$. Show that

$$\Delta = \frac{1}{d^2} \sum_{i=1}^{d^2} \mathrm{Ad}_{U_i}$$

for unitary operators $U_1, \ldots, U_{d^d}$ in $B(\mathcal{H})$. (Hint: Use the Weyl-Heisenberg basis for $B(\mathbb{C}^d)$.)