MAT4430, oral examination 1-2 June 2026

The curriculum consists of the material covered in class (lectures 1-30 in notes posted on the course webpage under Timetable), which is based on the compendium "Quantum Information Theory" by A. Müller-Hermes (lecture notes from 2025). Questions at the exam will be taken from the following list. You can expect to be asked questions from all three parts (I), (II), (III).

(I) Concepts and results (without proofs)

(1) Coding scheme for a classical distribution; discrete memoryless information source; classical entropy; relative entropy; convexity and concavity of entropies.
(2) Classical communication channel; coder and decoder; the capacity of a classical channel.
(3) Positive definite and positive semidefinite operators; characterizations of positivity; the trace; the Hilbert-Schmidt norm.
(4) Tensor product of operators; vectorization.
(5) Quantum states; pure states; the qubit.
(6) Measurements: POVM, PVM, von Neumann, destructive. POVM's with a fixed number of outputs.
(7) Entanglement; separable and entangled states; the partial transpose of the state of a composite system; the partial trace of a state of a composite system; marginal states.
(8) Correlations; deterministic and classical correlations; quantum correlations; Tsirelson's bound.
(9) Open quantum systems: purification of a quantum state; necklace identity; completely positive map; quantum channel.
(10) Representations of a quantum channel: the Choi matrix; Kraus decompositions; Stinespring dilations.
(11) The completely depolarizing channel and the depolarizing channel with a parameter p. Unitary channels.
(12) Instruments: non-destructive measurement; Naimark dilation of a POVM.
(13) The Bell basis: quantum teleportation.
(14) Classical mutual information; the accessible information of an ensemble formed of a probability distribution over an alphabet and a collection of quantum states (in a fixed complex Euclidean space) indexed over the alphabet.
(15) The von Neumann entropy of a quantum state; the quantum relative entropy of a pair of quantum states; properties of concavity and convexity; invariance under conjugation with an isometry; the data-processing inequality for the quantum relative entropy.
(16) Additivity and strong subadditivity of the von Neumann entropy.
(17) Classical-quantum states and their entropy; Holevo information of an ensemble; quantum mutual information; Holevo's theorem.
(18) Elementary properties of the von Neumann entropy and the quantum relative entropy;
(19) The fidelity function; invariance under conjugation with an isometry; Uhlmann's theorem.
(20) Coding scheme for classical information transmission over a quantum states; the classical capacity of a quantum state; the Holevo-Schumacher-Westmoreland theorem (capacity is bounded by the liminf of regularizations of the channel).

(21) Entanglement-breaking quantum channel. Additivity of Holevo capacity when one component is entanglement breaking.

(II) Results - with proofs

(1) The vectorization map preserves inner-products.
(2) The set of quantum states of the qubit is identified as the Bloch ball.
(3) The maximally entangled state is entangled.
(4) The CHSH inequalities for classical correlations.
(5) The existence of Kraus decompositions of a quantum channel.
(6) The existence of Stinespring dilations of a quantum channel.
(7) The Choi matrix of the completely depolarizing channel.
(8) An instrument for a given POVM.
(9) The relation between von Neumann entropy and quantum relative entropy.
(10) The von Neumann entropy is subadditive.
(11) The fidelity function on a pair of pure states.
(12) The data-processing inequality for the quantum relative entropy (assuming joint concavity of the entropy and existence of a Weyl-Heisenberg system).

(III) Examples

(1) Classical channels.
(2) Measurements.
(3) A quantum correlation that is not classical.
(4) Quantum channels.
(5) Kraus decomposition of quantum channels.
(6) Kraus decomposition of the fully depolarizing channel using a Weyl-Heisenberg system.