UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Exam in:	MAT3500/4500 — Topology
Day of examination:	Monday 08. December 2025
Examination hours:	9.00 – 13.00
This problem set consists of 2 pages.	
Appendices:	None
Permitted aids:	None

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

No justification is required in Problem 1. In Problems 2, 3, 4 and 5 all answers must be justified.

Problem 1 (weight 20%)

Complete the following definitions:

a (weight 4%)

A topological space X is *compact* if...

b (weight 4%)

Let X be a topological space and $(x_n)_{n\in\mathbb{N}}$ a sequence of points $x_n \in X$. The sequence x_n converges to $x \in X$ if...

c (weight 4%)

A nonempty topological space X is *connected* if...

d (weight 4%)

A topological space X is *normal* if...

e (weight 4%)

Let (X, d) be a metric space. A subset $U \subset X$ is open in the *metric topology* on X if...

(Continued on page 2.)

Problem 2 (weight 20%)

Recall that a subset $T \subset X$ of a topological space is *dense* if $\overline{T} = X$, where $\overline{T}$ denotes the closure of T.

Let (X, d) be a metric space, and $T \subset X$ a dense subset. Prove that the collection of subsets

$$\mathcal{B} = \{B_d(x, \tfrac{1}{n}) \mid n \in \mathbb{N}, x \in T\}$$

is a basis for the metric topology on X.

Problem 3 (weight 20%)

Let X be a topological space and S a set. Call a function $f : X \to S$ *locally constant* if every point $x \in X$ has a neighborhood U such that $f(U)$ consists of a single element. Prove that for a function $f : X \to S$ the following are equivalent:

1. $f : X \to S$ is locally constant.
2. $f : X \to S$ is continuous with respect to the discrete topology on S.

Problem 4 (weight 20%)

Let X be a Hausdorff space. Show that

$$\{(x, x) \mid x \in X\} \subset X \times X$$

is a closed subset of $X \times X$.

Problem 5 (weight 20%)

Recall that $D^n = \{x \in \mathbb{R}^n \mid \|x\| \leq 1\}$ and $S^n = \{x \in \mathbb{R}^{n+1} \mid \|x\| = 1\}$.

Consider the continuous function $f : D^n \to \mathbb{R}^{n+1}$ given by

$$f(x) = (2\sqrt{1 - \|x\|^2}x_1, \ldots, 2\sqrt{1 - \|x\|^2}x_n, 2\|x\|^2 - 1)$$

a (weight 10%)

Show that f corestricts to a continuous function $D^n \to S^n$.

b (weight 10%)

Show that $\widetilde{f}([x]) = f(x)$ defines a homeomorphism $\widetilde{f} : D^n/\!\sim \to S^n$, where the equivalence relation $\sim$ is defined by

$$x \sim y \text{ if } x = y \text{ or } x, y \in S^{n-1}.$$

— FIN —