

12<sup>th</sup> October, 2026

# MAT3500/4500

Mandatory assignment

**Submission deadline**

Thursday 29<sup>th</sup> October 2026, 14:30 in Canvas ([canvas.uio.no](https://canvas.uio.no)).

**Complete guidelines about delivery of mandatory assignments:**

[uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html](https://uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html)

GOOD LUCK!

Every submitted solution will be awarded a passing grade. Collaboration is permitted and encouraged. LLM assistance is permitted but strongly discouraged.

All submissions will receive detailed feedback.

**Problem 1.** Let  $(X, d)$  be a metric space.

- (a) Show that  $d : X \times X \rightarrow \mathbb{R}$  is continuous, where  $X \times X$  has the product topology.
- (b) Show that for  $x, y \in X$ , the subset  $A = \{z \in X \mid d(x, z) = d(y, z)\}$  of  $X$  is closed.

**Problem 2.** Let  $S = \mathbb{N} \cup \{\infty\}$ . Find a topology on  $S$  for which a function  $f : S \rightarrow X$  (for any topological space  $X$ ) is continuous if and only if the sequence  $f(n)$  converges to  $f(\infty)$  in  $X$ .

**Problem 3.** Let  $X, Y$  be topological spaces,  $f, g : X \rightarrow Y$  two continuous functions, and assume that  $f|_A = g|_A$  for a dense subset  $A \subset X$ .

- (a) Show that if  $Y$  is Hausdorff, then  $f = g$ .
- (b) Find an example which shows that this is not always true if  $Y$  is not Hausdorff.

**Problem 4.** Consider the following collection of subsets of  $\mathbb{R}$ :

$$\mathcal{B} = \{[a, \infty) \subset \mathbb{R} \mid a \in \mathbb{R}\}$$

- (a) Show that  $\mathcal{B}$  forms a basis for a topology  $\mathcal{T}'$  on  $\mathbb{R}$ .
- (b) Show that every continuous function  $f : (\mathbb{R}, \mathcal{T}') \rightarrow (\mathbb{R}, \mathcal{T})$  is constant, where  $\mathcal{T}$  denotes the usual topology on  $\mathbb{R}$ .

**Problem 5.** Let  $A = S^1 \cap \{(x, y) \in \mathbb{R}^2 \mid y \geq 0\}$ ,  $B = \{(x, y) \in \mathbb{R}^2 \mid x \in [-1, 1], y = 0\}$ , and  $X = A \cup B$  their union, with the subspace topology as a subspace of  $\mathbb{R}^2$ . Show that  $X$  is homeomorphic to  $S^1$ . (Be precise about continuity!)

**Problem 6.** Define an equivalence relation on  $\mathbb{R}$  as follows:

$$x \sim y \text{ iff } x - y \in \mathbb{Z}.$$

Write  $\mathbb{R}/\mathbb{Z}$  for the quotient  $\mathbb{R}/\sim$ . Show that  $\mathbb{R}/\mathbb{Z}$  is homeomorphic to  $S^1$ .

**Problem 7.** Let  $X = \prod_{n \in \mathbb{N}} \{-1, 1\} = \{-1, 1\}^{\mathbb{N}}$  with the product topology (where  $\{-1, 1\}$  has the discrete topology).

(a) For  $0 < q < 1$ , show that  $f : X \rightarrow \mathbb{R}$ ,

$$f((x_n)_{n \in \mathbb{N}}) = \sum_{n=1}^{\infty} x_n q^n$$

is continuous.

(b) For  $0 < q < \frac{1}{2}$ , show that  $f$  is injective.

(c) (Tricky bonus problem:) For  $0 < q < \frac{1}{2}$ , show that  $f$  is actually an embedding, by showing that  $f : X \rightarrow f(X)$  is open.